A CONSTRUCTION FOR A REGULAR POLYGON OF SEVENTEEN SIDES.

By Herbert W. Richmond, King's College, Cambridge.

Let OA, OB (fig. 6) be two perpendicular radii of a circle. Make OI one-fourth of OB, and the angle OIE one-fourth of OIA; also find in OA produced a point F such that EIF is 45°. Let the circle on AF as diameter cut OB in K, and let the circle whose centre is E and radius EK cut OA in N_3 and N_5; then if ordinates N_3P_3, N_5P_5 are drawn to the circle, the arcs AP_3, AP_5 will be $3/17$ and $5/17$ of the circumference.

Proof. Let C denote the angle OIE, so that $4C = OIA$, and $\tan 4C = 4$; also let α stand for $2\pi/17$.

Then (cf. Hobson's *Trigonometry*, p. 111)

$$2(\cos\alpha + \cos 2\alpha + \cos 4\alpha + \cos 8\alpha),$$

and

$$2(\cos 3\alpha + \cos 6\alpha + \cos 5\alpha + \cos 7\alpha)$$

are the roots of $z^2 + z = 4$, or of

$$z^2 + 4z\cot 4C = 4.$$

Therefore

$$2(\cos\ \alpha + \cos 2\alpha + \cos 4\alpha + \cos 8\alpha) = 2\tan 2C,$$

$$2(\cos 3\alpha + \cos 6\alpha + \cos 5\alpha + \cos 7\alpha) = -2\cot 2C.$$

Again, $2(\cos 3\alpha + \cos 5\alpha)$ and $2(\cos 6\alpha + \cos 7\alpha)$ are the roots of

$$x^2 + 2x\cot 2C = 1.$$

Therefore

$$2(\cos 3\alpha + \cos 5\alpha) = \tan C,$$

$$2(\cos 6\alpha + \cos 7\alpha) = -\cot C.$$

Similarly,

$$2(\cos\ \alpha + \cos 4\alpha) = \tan(C + 45°),$$

$$2(\cos 2\alpha + \cos 8\alpha) = \tan(C - 45°).$$

Finally, we may write the results in the following form:

$$\left.\begin{aligned}
2\cos 3\alpha + 2\cos 5\alpha &= 2\cos\ \alpha \,.\, 2\cos 4\alpha = \tan C \\
2\cos\ \alpha + 2\cos 4\alpha &= 2\cos 6\alpha \,.\, 2\cos 7\alpha = \tan(C + 45°) \\
2\cos 6\alpha + 2\cos 7\alpha &= 2\cos 2\alpha \,.\, 2\cos 8\alpha = \tan(C + 90°) \\
2\cos 2\alpha + 2\cos 8\alpha &= 2\cos 3\alpha \,.\, 2\cos 5\alpha = \tan(C - 45°)
\end{aligned}\right\} \quad (A).$$

Now, by the construction given above,

$$2\cos P_3OA + 2\cos P_5OA = 2\frac{ON_3 - ON_5}{OA} = 4\frac{OE}{OA} = \frac{OE}{OI} = \tan C,$$

$$2\cos P_3OA \,.\, 2\cos P_5OA = -4\frac{ON_3 \,.\, ON_5}{OA^2} = -4\frac{OK^2}{OA^2} = -4\frac{OF}{OA}$$

$$= -\frac{OF}{OI} = \tan(C - 45^\circ).$$

Hence $P_3OA = 3\alpha$ and $P_5OA = 5\alpha$.

Many other geometrical constructions are suggested by the equation (A); the one selected seems to lead to the most compact figure.

It may be similarly shewn that, if $2C$ be the acute angle whose tangent is 2, and α stand for $2\pi/5$, then will

$$2\cos\alpha = \tan C, \text{ and } 2\cos 2\alpha = -\cot C.$$

Therefore (fig. 7), if OA, OB be two perpendicular radii of a and J the middle point of OB; then the ordinates drawn circle, through E and F, the points where the bisectors of OJA meet OA, determine on the circle four points which with A form a regular pentagon.

ON THE GENERAL EQUATION OF A CONICOID THAT HAS DOUBLE CONTACT WITH TWO GIVEN CONICOIDS.

By A. C. Dixon, Fellow of Trinity College, Cambridge.

TAKE the equations of the two given surfaces in the form

$$S_1 \equiv a_1x^2 + b_1y^2 + c_1z^2 + d_1u^2 = 0,$$
$$S_2 \equiv a_2x^2 + b_2y^2 + c_2z^2 + d_2u^2 = 0.$$

Then the equation sought may be written in either of the following forms:

$$S_1 + (l_1x + m_1y + n_1z + r_1u)(\lambda_1x + \mu_1y + \nu_1z + \rho_1u) = 0,$$
$$S_2 + (l_2x + m_2y + n_2z + r_2u)(\lambda_2x + \mu_2y + \nu_2z + \rho_2u) = 0.$$

Since these two equations are equivalent,

$$\frac{l_1\mu_1 + \lambda_1m_1}{l_2\mu_2 + \lambda_2m_2} = \frac{l_1\nu_1 + \lambda_1n_1}{l_2\nu_2 + \lambda_2n_2} = \dots .$$