

On a relationship between record times and record values of an i.i.d. sequence

Let $\{X_n\}$ be an i.i.d. sequence of random variables with a continuous c.d.f. F . Define upper and lower record times by

$$\begin{aligned} U_0 &= 1, & U_{n+1} &= \inf \{k; X_k > X_{U_n}\}; & n &\geq 0; \\ L_0 &= 1, & L_{n+1} &= \inf \{k; X_k < X_{L_n}\}; & n &\geq 0. \end{aligned}$$

If F is the c.d.f. of an exponential distribution with unit mean, then we have the following result.

- Theorem.* (a) L_n and $L_n X_{L_n}$ are independent for all $n \geq 0$.
(b) $L_n X_{L_n}$ is exponentially distributed with unit mean.

This theorem allows the following conclusion.

Corollary. (a) $X_{U_n} - \log U_n$ is asymptotically Λ -distributed for $n \rightarrow \infty$ where Λ denotes the c.d.f. of a doubly-exponentially distributed random variable.

$$(b) \log U_n = X_{U_n} + O(\log n) \text{ a.s. } (n \rightarrow \infty).$$

Similar results can be derived also for the case of a general c.d.f. F , as well as strong approximations jointly for record times, inter-record times and record values by Poisson and Wiener processes.