

Modeling the redundant signals effect by specifying the hazard function

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In the bimodal detection task, the observer must respond as soon as a signal is presented in either of two modalities (e.g., a tone or a flash). On *single-signal* trials, only one signal is presented; on *redundant-signal* trials, both signals are presented either simultaneously or with a short stimulus onset asynchrony (SOA). The common empirical finding is that response time (RT) is shorter for redundant-signal trials. A number of sensory processing models have been proposed for this *redundant-signals effect* (RSE). These models differ in terms of assumptions made with respect to the underlying stochastic mechanisms (e.g., Meijers & Eijkman, 1977; Nickerson, 1973; Raab, 1962).

Recently, special efforts have been made to derive empirically testable properties from various models (e.g., Colonius, 1986; Diederich & Colonius, 1987; Gielen, Schmidt, & Van den Heuvel, 1983; Miller, 1982; Ulrich & Giray, 1986). In this note, we propose a particular way of dealing with some of these models in an integrated manner by using the concept of a *hazard function*. The hazard function is very closely related to the probability distribution (see definition below). It specifies, for any point in time, the tendency for the response to occur instantaneously given that it has not yet occurred. It will be shown that formulating models for the RSE in terms of the hazard function clarifies their underlying assumptions and facilitates derivation of their testable properties. This will be demonstrated with reference to a recent paper by Miller (1986), in which a number of different models for the RSE are discussed.

The hazard function, a probabilistic concept originally developed in reliability theory (cf. Barlow & Proschan, 1975), has become a valuable tool in the analysis of RTs (see, e.g., Bloxom, 1984, 1985; Luce, 1986, p. 13ff; Townsend & Ashby, 1983).

DEFINITION. Let T be a nonnegative real valued random variable with probability distribution function $F(t)$ such that the corresponding density function $f(t)$ exists. The hazard function is defined by

$$h(t) = f(t)/(1-F(t)) \quad (1)$$

for all t for which $F(t) < 1$.

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According to this definition, the hazard function at time t is just the density renormalized by the probability that the response failed to occur prior to t , that is, $1 - F(t)$. Model assumptions formulated in terms of the hazard function can easily be translated into conditions on the corresponding probability distribution of response times by noting the following relationship (see, e.g., Barlow & Proschan, 1975, p. 53), which follows from integrating both sides of Equation 1:

$$F(t) = 1 - \exp\left[-\int_0^t h(s) ds\right]. \quad (2)$$

In the following, we first demonstrate our approach on the well-known independent probability summation model; later, the so-called "exponential coactivation" model proposed by Miller (1986) will be dealt with.

Let V , A , R be the RT if a single visual, a single auditory, and a simultaneous double (redundant) stimulus are presented, respectively. For the corresponding RT probability distribution functions, let us write F_V , F_A , and F_R . The corresponding hazard rates are denoted h_V , h_A , and h_R . For the rest of this paper, we assume an experimental situation where a visual stimulus is followed by an auditory stimulus d msec later ($d \geq 0$). The corresponding probability distribution of RTs is denoted by $F_R(t;d)$ (note that, in particular, $F_R(t) = F_R(t;0)$).

Independent Probability Summation Model

It is assumed that each stimulus presented generates a neural activity that may trigger the response mechanism. If two stimuli are present, the response is caused by the first process to finish (hence, the name "race models" sometimes used). Thus, in the redundant-signal situation, if the processes do not influence each other, their tendencies to elicit a response instantaneously should add up to an overall tendency to terminate the next instant of time. Taking into account that the acoustic stimulus is presented d msec later, this situation translates into the following hazard function for a redundant-signal trial:

$$h_{R,d}(t) = \begin{cases} h_V(t) & \text{if } t \leq d \\ h_V(t) + h_A(t-d) & \text{if } t > d. \end{cases} \quad (3)$$

In order to derive this model's prediction for the distribution of RTs in the redundant-signal situation, we have to insert the above hazard function into Equation 2. This yields, after some simple algebraic manipulation (see Appendix), the familiar expression for the independent probability summation model:

$$F_R(t;d) = \begin{cases} F_V(t) & \text{if } t \leq d \\ 1 - [1 - F_V(t)] * [1 - F_A(t-d)] & \text{if } t > d \end{cases} \quad (4)$$

Exponential Coactivation Model

This class of models proposed by Miller (1986) also assumes a different response tendency for each signal type: acoustic (*A*) visual (*V*), or simultaneous acoustic/visual (*R*) signal present. The intuition for the exponential model is as follows:

Consider a trial on which a visual signal is presented first and an auditory signal is presented after some nonzero SOA_A. According to exponential models, a response observed on this trial will have been activated either by the visual signal present before the end of the SOA_A or by the redundant signals present after the end of the SOA_A. (Miller, 1986, p. 333)

By a rough analogy with the race model, Miller then came up with the following inequality for the corresponding probability distributions:

$$F_R(t;d) \leq F_V(t) + F_R(t-d) \text{ for all } t. \quad (5)$$

It is possible, however, to derive a much stronger relation between these distributions if exponential coactivation is to hold. Obviously, the hazard function for exponential coactivation models reads as follows:

$$h_{R,d}(t) = \begin{cases} h_V(t) & \text{if } t \leq d \\ h_R(t-d) & \text{if } t > d. \end{cases} \quad (6)$$

Note, in particular, that $h_{R,0}(t) = h_R(t)$; inserting the above hazard function into Equation 2 yields (see Appendix)

$$F_R(t;d) = \begin{cases} F_V(t) & \text{if } t \leq d \\ 1 - [1 - F_V(d)] * [1 - F_R(t-d)] & \text{if } t > d. \end{cases} \quad (7)$$

Thus, an explicit expression for the response time distribution of the exponential coactivation model is available and can be used for testing the model. Moreover, since both sides of Equation 7 are probability distributions, standard Kolmogorov-Smirnov tests should be applicable. Of course, Equation 7 implies Miller's Inequality 5.

Exponential coactivation models are history-free in the following sense. By the end of the SOA ($t=d$), if a response has not yet occurred (triggered by the visual signal alone), the tendency to respond in the next instant of time is reset to the level corresponding to a simultaneous redundant-signal trial, that is, with an SOA equal to zero. Miller (1986) found response times too fast to satisfy Inequality 5, and was thus led to consider models assuming accumulation of activation over time, as proposed by Grice, Canham, and Boroughs (1984). In their model, response activation grows deterministically as a function of time since onset of the signal. When response activation reaches a criterion, the response is initiated. Variation in RT is caused by the fluctuation of this response criterion, which is assumed to have a normal distribution with unknown mean and variance. Although derivation of the hazard function—or of closely related quantities—is pos-

sible for this type of model too, it does not seem to lead to any simple predictions that would allow a feasible test of the model (see, e.g., Luce, 1986, p. 151).

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APPENDIX

The *independent probability summation model* is defined by the hazard rate

$$h_{R,d}(t) = \begin{cases} h_V(t) & \text{if } t \leq d \\ h_V(t) + h_A(t-d) & \text{if } t > d. \end{cases} \quad (3)$$

It follows from Equation 2 that

$$1 - F_R(t;d) = \exp \left[- \int_0^t h_{R,d}(s) ds \right];$$

Inserting Equation 3 implies, for $t > d$,

$$\begin{aligned} 1 - F_R(t;d) &= \exp - \left[\int_0^d h_V(s) ds + \int_d^t (h_V(s) + h_A(s-d)) ds \right] \\ &= \exp - \left[\int_0^t h_V(s) ds + \int_d^t h_A(s-d) ds \right]; \end{aligned}$$

a change of variables in the last integral, $s' = s-d$, yields

$$\begin{aligned}
 &= \exp - \left[\int_0^t h_V(s) ds + \int_0^{t-d} (h_A(s')) ds' \right] \\
 &= [1 - F_V(t)] * [1 - F_A(t-d)].
 \end{aligned}$$

The *exponential coactivation model* is defined by the hazard rate

$$h_{R,d}(t) = \begin{cases} h_V(t) & \text{if } t \leq d \\ h_R(t-d) & \text{if } t > d. \end{cases} \quad (6)$$

It follows from Equation 2 that

$$1 - F_R(t;d) = \exp \left[- \int_0^t h_{R,d}(s) ds \right];$$

Inserting Equation 6 yields for, $t \leq d$,

$$\begin{aligned}
 1 - F_R(t;d) &= \exp \left[- \int_0^t h_V(s) ds \right]; \\
 F_R(t;d) &= F_V(t).
 \end{aligned}$$

Inserting Equation 6 yields, for $t > d$,

$$1 - F_R(t;d) = \exp - \left[\int_0^d h_V(s) ds + \int_d^t h_R(s-d) ds \right];$$

changing variables, $s' = s-d$, in the last integral, we have

$$\begin{aligned}
 1 - F_R(t;d) &= \exp - \left[\int_0^d h_V(s) ds + \int_0^{t-d} h_R(s') ds' \right]; \\
 &= [1 - F_V(d)] * [1 - F_R(t-d)]; \\
 F_R(t;d) &= 1 - [1 - F_V(d)] * [1 - F_R(t-d)].
 \end{aligned}$$

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