Carl v. Ossietzky
UNIVERSITÄT
OLDENBURG

Wirtschaftswissenschaftliche Diskussionspapiere

Altruism, redistribution and social insurance

Udo Ebert and Oskar von dem Hagen

V-166-96

August 1996

Volkswirtschaftliche Reihe
Institut für Volkswirtschaftslehre I
Universität Oldenburg, D-26111 Oldenburg

Altruism, redistribution and social insurance*

Udo Ebert and Oskar von dem Hagen

Department of Economics

University of Oldenburg

August 1996

*The authors would like to acknowledge helpful comments by Richard Arnott, Robin Boadway and Dan Usher. Of course, we take full responsibility for any remaining errors.

1 Introduction

Why do governments redistribute income? Why do individuals voluntarily redistribute income? Is one of these modes of redistribution better than the other? The present paper attempts some answers to these questions.

It is difficult to assess the actual amount of redistribution by governments, partly because of lack of data, partly because the redistributive effect of public provision of private and especially public goods is largely unknown. However, as a first approximation, the comparison of the average tax rate may serve as an indicator of relative importance attributed to redistribution by the government. A comparison of the US and Europe shows that redistributive taxation is more important in Europe—data for Germany may serve as representative. The Statistisches Bundesamt (1995) reports an average tax rate (including social security) of 29% of GNP for the US in 1980, and of 38% for Germany. It will be shown in this paper that redistribution and social insurance are closely related, in fact, they may be regarded as substitutes. When one focuses on social security, the difference between the US and Europe is even more pronounced. Government expenditures for social security (including unemployment benefits, pensions and health services) amounted to 6.8% of GDP in the US in 1980, and to 18.6% in Germany. On the other hand, voluntary transfers like charitable contributions or donations play a larger role in the US. Paqué (1986) reports private donations of 2.27% (1.23%) of NDP for the US and 1.06% (0.24%) for Germany for 1980. The percentages in brackets exclude religious donations.

These observations confirm the rather natural assumption that government redistribution ‘crowds out’ private redistributive transfers. But the questions remain why these systems are different and whether there are theoretical reasons to prefer one over the other.

These question are examined in a very simple model: Two identical individuals will, by chance, be assigned two jobs of different productivity. Ex post the individuals are different because they receive different wages. Under what conditions would they agree on transfer payments? If one considers ex ante or ex post agreement on transfer schedules with or without altruism, then there are four major cases one has to look into. Only lump-sum taxes or transfers will be considered.

The formalization of altruism is very basic: The minimum consumption of an individual enters the utility function as a pure public good. Furthermore, the altruism term is linear. This implies that the marginal benefit of a donation is constant and independent of the distance between the donor's and the recipient's wage income. In reality, of course, altruism is much more complex.

However, despite its simplicity, the formalization grasps several aspects of altruism:

- It is not the donation by itself that generates a 'warm glow', but the donation to a specific recipient. This is sometimes referred to as 'pure altruism' (cf. Smith et al., 1995).
- The criterion by which a recipient is chosen has to be observable—this excludes the recipient's well-being or his utility level.
- Using income as the criterion for donations creates an incentive problem with the recipient, who can increase his consumption of leisure as a reaction to a donation—and he will do so if leisure is a normal good.

The public good will be of the form (wage·labour+transfer) and thus both individuals can supply it, the low-wage individual by increasing his labour supply and the high-wage individual by increasing his donation. In many aspects this formulation corresponds to the usual models of voluntary public-good provision. Both individuals ignore the external benefit of their contribution and both can react to an increase in provision by reducing their contribution. However, while this behaviour of the recipient does lead to Pareto inefficiency, in the donor's case it does not.

One problem not considered in this paper is the one concerning several donors. In that case the voluntary donations generally lead to an inefficient level of donation because the positive externalities of the public good *with respect to the other donors* are ignored (see Arrow, 1981). These inefficiencies could be overcome by using allocation mechanisms for public goods like the MDP procedure.

The simplicity of the model exhibits a fundamental difference between ex ante and ex post transfer schedules. Ex post, the individuals and the objectives of the donor and the recipient are different. In general, there will be an infinity of Pareto-optimal transfers but typically only one of these will be individually optimal for the high-wage individual.

The ex ante approach with redistributive taxes—and little or no voluntary transfers—may be referred to as the 'European' or fiscal solution, while the laissez-faire approach of voluntary transfers resembles the 'American' approach to redistribution. In the present paper there are two individuals who consider ex ante an optimal tax schedule. Since they face the same probabilities, their objectives are identical and there will be a single Pareto optimum. However, the story can easily be extended to a legislative assembly pondering a just tax schedule for future generations. The ex ante solution will, generally, not be self-enforcing. I.e. the government by its coercive power must implement the agreed tax/transfer schedule because it would be in the interest of some (high-wage) individuals to pay less taxes even if they were altruistic.

The approach chosen in this paper differs considerably from the literature on altruism, optimal redistribution and insurance (beginning with Hochman and Rodgers, 1969, Arrow, 1981, Becker, 1981, and Varian, 1980). Firstly, altruism is based on minimal income or a comparison of incomes in society. The minimum income can be interpreted as a public good in the spirit of Thurow (1971). Both individuals are able to contribute to its provision. It is *not* the level of utility of the recipient or donor which is taken into account. Secondly, the framework is not characterized by reciprocity. Only the income of the poor individual does play a role. Both individuals' altruism focuses on minimal income. That can, thirdly, be understood as an indicator of the income distribution or of income inequality. It obviously represents only a partial indicator—but, in this respect the model is comparable to Arrow (1981). Fourthly, the focus on income as the subject of altruism distinguishes this paper from the large body of literature on merit wants and paternalistic preferences. Finally, labour supply is taken into consideration explicitly. This allows incentive effects of donations to be described and a discussion of their impact on the labour-leisure choice to be incorporated.

The organization of the paper is as follows: The exposition of the model is followed by section 3, which examines the differences between the European and the American approach in the absence of altruism. The subsequent section investigates the changes that altruism induces in individual behaviour. Both sections begin with a model where labour supply is fixed. This allows the incentive problem due to variable labour supply to be identified later on.

2 The Model

The wage rates of the two jobs will be denoted by w_1, w_2 and, without loss of generality, it will be assumed that $w_1 < w_2$. The jobs are assigned with probability one half to two identical individuals. There is one consumption good C and labour L. The preferences of the individuals are represented by a von Neumann-Morgenstern utility function over C and L:

$$u^i = u[C^i, L^i] + \alpha \min_{j=1,2} C^j. \tag{1}$$

C^i is the consumption of person i and L^i is his labour supply. If $\alpha > 0$, then the individuals are altruistic, otherwise α is zero. The non-altruistic utility function (when $\alpha = 0$) is assumed to satisfy

$$u_C > 0,\ u_L < 0, \tag{2}$$

$$u_{CC},\ u_{LL} < 0,\ u_{CC}u_{LL} - u_{CL}^2 > 0, \tag{3}$$

$$-u_L u_{CC} + u_C u_{LC} < 0 \tag{4}$$

$$-u_L u_{CL} + u_C u_{LL} < 0. \tag{5}$$

The different conditions require the marginal utilities of consumption and leisure to be positive (2), the marginal utility of labour thus has to be negative. The second set of conditions imposes concavity (3) and the last two are equivalent to leisure (4) and consumption (5) being normal goods. However, consumption and leisure could be substitutes ($u_{CL} > 0$) or complements ($u_{CL} < 0$).

The person with the low-wage job will be called individual 1. T denotes the transfer from the high-income to the low-income individual. C is used as numéraire so that the individual budget constraints are

$$C^1 = w_1 L^1 + T \tag{6}$$

$$C^2 = w_2 L^2 - T. \tag{7}$$

Given these properties, it is straightforward to show that

- Voluntary transfers, whether unilateral ex post or agreed upon ex ante, will always be from the high-wage to the low-wage individual.

- In the absence of altruism and of transfers, the high-wage individual will receive a higher income, consume more, and his marginal utility of consumption will be less than that of the low-wage individual. The relative size of the labour supply, however, is indeterminate.

3 No altruism

As a reference scenario it is useful to examine a situation where labour supply is fixed at $L^i = \bar{L}$. This will allow the relationship between voluntary contributions, Pareto optimality and, later on, altruism in the given framework to be clarified.

Ex post each individual knows his wage. Given that labour supply is fixed, individual 1 has nothing to decide. Individual 2 will choose a transfer $T \geq 0$ to

$$\max_T u^2 = u[w_2\bar{L} - T, \bar{L}].$$

The solution implies that he makes no voluntary payments ($T = 0$) and, obviously, is Pareto efficient. For future reference we will define the resulting relative valuation as

$$\bar{g} \equiv \frac{u_c^1[w_1\bar{L}, \bar{L}]}{u_c^2[w_2\bar{L}, \bar{L}]}. \tag{8}$$

Ex post, individual 2 unilaterally decides his voluntary transfer. What would happen if the individuals decided about the transfer ex ante? They could formulate a social decision problem, and there would be no aggregation problem since, ex ante, the individuals are identical. Any ex ante agreement about ex post transfers, on the other hand, will be violated by the high-wage individual, if it is not binding. His ex post choice of a transfer would be zero.

Proposition 1

If the individuals are not altruistic, labour supply is fixed and the transfer is decided on ex post, then

a) no voluntary transfer is made and

b) the outcome is Pareto efficient.

The set of ex post Pareto optima can be derived by maximizing an individualistic social welfare function:

$$\max_T \mathcal{W} = (1-\gamma)u[w_1\bar{L} + T, \bar{L}] + \gamma u[w_2\bar{L} - T, \bar{L}],$$

where γ $(0 \leq \gamma \leq 1)$ denotes the weight of the high-wage individual. This leads to the necessary condition

$$\frac{u_C^1}{u_C^2} = \frac{\gamma}{1-\gamma}.$$

Varying γ between zero and one, traces out all Pareto optima including the case where

$$\gamma = \frac{\bar{g}}{1+\bar{g}},$$

and where $\bar{g}$ is as defined in (8). Since $\bar{g}$ is positive, the resulting γ lies between zero and one. This proves, in particular, that the voluntary transfer of $T = 0$ is Pareto optimal.

It is reasonable, however, to assume that the two individuals could *ex ante* make binding agreements on ex post transfers by determining a tax schedule which the government is allowed to enforce. In the presence of fixed labour supply $\bar{L}$, the individual and social optimization problem is to maximize the expected utility of the representative individual:

$$\max_T \frac{1}{2} u[w_1 \bar{L} + T, \bar{L}] + \frac{1}{2} u[w_2 \bar{L} - T, \bar{L}]. \tag{9}$$

The solution can be characterized by

Proposition 2

If the individuals are not altruistic, labour supply is fixed and the transfer is decided on ex ante, then the unanimously agreed upon tax schedule will lead to identical consumption and utility levels for both individuals.

This follows from the first-order condition

$$u_C^1 = u_C^2.$$

Since labour supply is fixed and $u_{CC} < 0$, we get

$$T = (w_2 - w_1)\bar{L}/2$$

$$C^1 = C^2$$

$$u^1 = u^2$$

$$-u_L^1 = -u_L^2.$$

A risk-averse individual will always prefer the certain utilitity of the expected income to the expected utility of the uncertain income. Thus, ex ante both individuals unanimously prefer the tax solution to the situation with voluntary transfers. This comparison of the European fiscal solution and the American laissez-faire solution is incomplete, however, because in a market economy insurance companies might form that could provide a similar or even identical protection as in the fiscal solution.

An insurance company could provide insurance V against the situation of an inferior job at the actuarily fair price of 1/2. If the individual gets the good job, he only pays the insurance premium $V/2$. In case of the bad job he pays the insurance premium $V/2$ and receives the insurance payment V. The net receipt in this case is $V/2$. The objective of an individual is thus

$$\max_V \frac{1}{2}u[w_1\bar{L} + \frac{1}{2}V, \bar{L}] + \frac{1}{2}u[w_2\bar{L} - \frac{1}{2}V, \bar{L}].$$

Obviously, the problem is identical to (9) and the solution $V = (w_2 - w_1)\bar{L}$ of full insurance duplicates the fiscal solution.[1]

The problem becomes more complicated if labour supply is variable. In the context of voluntary *ex post* transfers, the individuals face a game situation where both can provide the public good—as opposed to a simple optimization problem of the high-wage individual as above. The usual solution concept would be the Nash equilibrium. However, since the problem is clearly asymmetric, a solution where the high-wage individual is a Stackelberg leader may be more appropriate. As regards Pareto optima and the utility possibility frontier, the solution may be different if the individuals (can be forced to) work the socially optimal amount of time or if they can choose their labour supply according to their own preferences.

In the absence of altruism the solution is both straightforward and trivial. The high-wage

[1] One problem with the fiscal and insurance solution is omitted by the specification of the model: The wages are exogenous. If it were costly in terms of money or labour to get a better job, a moral hazard problem would arise. Full insurance, whether by the government or an insurance company, would generate wrong incentives.

individual has no incentive to share his income. Maximization

$$\max_{L^2,T} u^2 = u[w_2L^2 - T, L^2]$$

leads to

$$\frac{-u_L^2}{u_C^2} = w_2 \text{ and } T = 0.$$

Thus, mutatis mutandis, proposition 1 still holds.

The characterization of the Pareto optima is straightforward, too. Since the individual labour supplies generate no externality and the transfer is lump sum, the individuals will choose a socially optimal amount of labour supply for any given transfer. Formally:

$$\max_{L^1,L^2,T} (1-\gamma)u[w_1L^1 + T, L^1] + \gamma u[w_2L^2 - T, L^2]$$

yields

$$-u_L^1/u_C^1 = w^1 \tag{10}$$

$$-u_L^2/u_C^2 = w^2 \tag{11}$$

$$u_C^1/u_C^2 = \frac{\gamma}{1-\gamma}.$$

The utility possibility frontier coincides with the set of Pareto optima and includes the solution with voluntary transfers $T = 0$.

Next we consider the situation where both individuals decide *ex ante* on the transfer. Initially it is supposed that the individuals' types can be observed, i.e. the transfer can be based on the wage-type. Afterwards we will discuss the same problem if it must be based on observed income, i.e. if information is incomplete.

If labour supply is variable, the easy solution derived above vanishes because the individuals have different leisure-consumption tradeoffs. However, in the absence of altruism the ex post optimization problems are uncorrelated and the individuals will choose the socially optimal amount of labour. The optimal tax can thus be determined from

$$\max_{L^1,L^2,T} \frac{1}{2}u[w_1L^1 + T, L^1] + \frac{1}{2}u[w_2L^2 - T, L^2].$$

It is described in

Proposition 3

If the individuals are not altruistic, labour supply is variable and the transfer is decided on ex ante, then

a) *the transfer is positive in general,*

b) *the resulting allocation can be characterized by*

$$L^1 < L^2$$

$$C^1 \begin{cases} = C^2, & \text{if } u_{CL} = 0 \\ < C^2, & \text{if } u_{CL} > 0 \\ > C^2, & \text{if } u_{CL} < 0 \end{cases} \text{ and} \tag{12}$$

c)

$$u^1 > u^2 \quad \text{if} \quad u_{CL} = 0 \quad \text{or} \quad u_{CL} < 0$$

$$u^1 \gtreqless u^2 \quad \text{if} \quad u_{CL} > 0.$$

The first-order conditions correspond to (10) and (11). Since $w_1 < w_2$ by assumption this implies

$$-u_L^1 < -u_L^2.$$

First, we would like to note that it cannot be optimal for individual 1 to work more than individual 2 because a switch in working hours between the individuals would allow a switch in consumption as well and generate some additional income. Even though $L^1 \leq L^2$ is a necessary condition for an optimum, there are still several cases possible as far as $C^1 \gtreqless C^2$ is concerned.

The first case of (12) follows immediately from (11). If $u_{CL} > 0$, i.e. if consumption and leisure are substitutes, we can write

$$u_C[C^1, L^2] > u_C[C^1, L^1] = u_C[C^2, L^2].$$

It then follows that $C^1 < C^2$. The third case can be derived analogously.

It is interesting to note that the low-wage individual will obtain ex post a higher utility level in cases one and three. In case two it is unclear which individual will receive the higher utility level.

The simple framework of proposition 3 sets the tone for the following: allowing labour supply to vary invalidates the full-insurance solution. In fact, it is not clear what 'full-insurance' would mean in the present context. With fixed labour supply the loss of income due to a bad job is well defined: $(w_2 - w_1)\bar{L}$. If it is variable, the actual loss depends on the labour supply.

However, as in the case with fixed labour supply, an insurance market would lead to the same optimum as the fiscal solution because, again, the optimization problems are formally identical with $V = 2T$.

In the context of two individuals and two given jobs, it can be assumed that the wage type is observable and the tax schedule, thus, can be based on it. In a more general framework with more than two jobs this assumption is less plausibleSee Stiglitz (1987) for a short discussion of this issue.. If the wage-type is unobservable, the corresponding risk is uninsurable by private companies. The fiscal solution would still be feasible but may be incentive incompatible. If the wage rate is unobservable, the tax payment can only be based on gross income. It may then be advantageous for a high-wage individual to pretend to be of the low-wage type (by earning the same income) and to receive social insurance payments.

To be incentive compatible, the fiscal solution faces the additional restriction that misrepresentation of the wage type must not be advantageous. This is considered in the following.

Let $Y^i = w_i L^i$, then the ex ante optimization problem

$$\max_T \frac{1}{2}u[Y^1 + T, \frac{Y^1}{w_1}] + \frac{1}{2}u[Y^2 - T, \frac{Y^2}{w_2}],$$

is subject to the constraint

$$u[Y^2 + T, \frac{Y^2}{w_2}] \geq u[Y^1 - T, \frac{Y^1}{w_2}],$$

and $Y_1 = Y_1[T]$ and $Y_2 = Y_2[T]$ are chosen ex post to maximize individual utility. For the low-wage type it is never advantageous to misrepresent his type. Thus there is only one incentive constraint to be observed.

In this case we obtain a solution we expect:

Proposition 4

If the individuals are not altruistic, labour supply is variable, and the transfer is decided on ex ante and is based on income, then

a) the transfer is positive in general and

b) the resulting allocation can be characterized by

$$C^1 < C^2 \qquad \text{and} \qquad u^1 < u^2.$$

It is not necessary to solve the optimization problem explicitly since the relevant properties of a solution can be derived by the same methodology used above.

Assume $Y^1 \geq Y^2$. Then

$$C_1 = Y^1 + T \geq Y^2 - T = C_2 \quad \text{for all} \quad T \geq 0.$$

Observing that $w_1 < w_2$, yields

$$L_1 = \frac{Y^1}{w_1} \geq \frac{Y^2}{w_1} > \frac{Y^2}{w_2} = L_2.$$

The conditions for utility maximization and normality lead to a contradiction. Therefore $Y^1 < Y^2$. Now suppose that $Y^1 + T \geq Y^2 - T$. We derive

$$u[Y^2 - T, \frac{Y^2}{w_2}] \leq u[Y^1 + T, \frac{Y^2}{w_2}] < u[Y^1 + T, \frac{Y^1}{w_2}],$$

which contradicts the incentive constraint.

Thus we obtain

$$C^1 = Y^1 + T < Y^2 - T = C^2.$$

On the other hand, the incentive constraint has to be satisfied:

$$u[C^2, \frac{Y^2}{w_2}] \geq u[C^1, \frac{Y^1}{w_2}].$$

Since $w_1 < w_2$ we have

$$u[C^2, \frac{Y^2}{w_2}] > u[C^1, \frac{Y^1}{w_1}].$$

Obviously the donor is *always* better off than the recipient. This solution is constrained Pareto-efficient. In this case the 'European' solution dominates the 'American' one because private insurance companies cannot insure an unobservable risk.[2]

4 Altruism

If the individuals are altruistic, the structure of the model changes. Now the minimum consumption is a pure public good in the sense that it is 'consumed' by both individuals.

In the reference situation with fixed labour supply and *ex post* voluntary transfers, only the high-wage individual can change the public-good provision by a transfer to the low-wage individual. Since he ignores that individual's benefit from this transfer, one might conjecture that the transfer will be too low. This conjecture, however, is not correct.

Individual 2 maximizes his utility by choosing a transfer T. The transfer has to be nonnegative and will never exceed

$$\bar{T}_{max} \equiv (w_2 - w_1)\bar{L}/2$$

because with this transfer both consumption levels are equal. One way to formulate the objective of individual 2 is to maximize his utility function (1) subject to his budget

[2] This also precludes the 'intermediate' policy of mandated benefits (cf. Summers, 1989).

constaint (7) and subject to $C^1 \leq C^2$. Using the budget restrictions (6) and (7) to eliminate C^i, the problem becomes

$$\max_{T,\lambda} u[w_2\bar{L} - T, \bar{L}] + \alpha(w_1\bar{L} + T) + \lambda(w_2\bar{L} - T - w_1\bar{L} - T).$$

The first-order conditions

$$-u_C^2 + \alpha - \lambda \leq 0,\ T(\cdot) = 0,\ T \geq 0,\ (w_2 - w_1)\bar{L} - 2T \geq 0,\ \lambda(\cdot) = 0,\ \lambda \geq 0$$

can give rise to different solutions, depending on the size of α. Define

$$\underline{\alpha} \equiv u_C[w_2\bar{L}, \bar{L}], \text{ and}$$

$$\overline{\alpha} \equiv u_C[(w_1 + w_2)\bar{L}/2, \bar{L}].$$

Proposition 5

If the individuals are altruistic and labour supply is fixed at $L^i = \bar{L}$, three different solutions exist for the transfer, depending on the size of α.

a)

$$T \begin{cases} = 0, & \text{if } \alpha \leq \underline{\alpha} \\ \text{satisfies } u_C[w_2\bar{L} - T, \bar{L}] = \alpha, & \text{if } \underline{\alpha} < \alpha < \overline{\alpha} \\ = T_{max}, & \text{if } \alpha \geq \overline{\alpha} \end{cases}$$

b) In every case the voluntary transfer is Pareto optimal.

The first part follows from the first-order condition. Since individual 2 chooses T to maximize his utility, it is not possible to increase his utility by a variation in T. Given T, individual 1's utility level is fixed—and thus maximal. The result, therefore, is Pareto optimal.

These results are illustrated in figure 1.[3] There, several utility possibility curves are drawn, corresponding to different levels of α. In the present context, the graph is symmetric

[3] The graph is a plot of the points (the leisure term is constant and set to zero)

$$\{\ln[w_1\bar{L} + T + 1] + \alpha \min[w_1\bar{L} + T, w_2\bar{L} - T], \ln[w_2\bar{L} - T + 1] + \alpha \min[w_1\bar{L} + T, w_2\bar{L} - T]\}$$

for $w_1 = 1$, $w_2 = 2$, $\bar{L} = 4$, $-w_1\bar{L} \leq T \leq w_2\bar{L}$ and values of α between zero and 0.5. With these parameters $\underline{\alpha} = 1/9$ and $\overline{\alpha} = 1/7$.

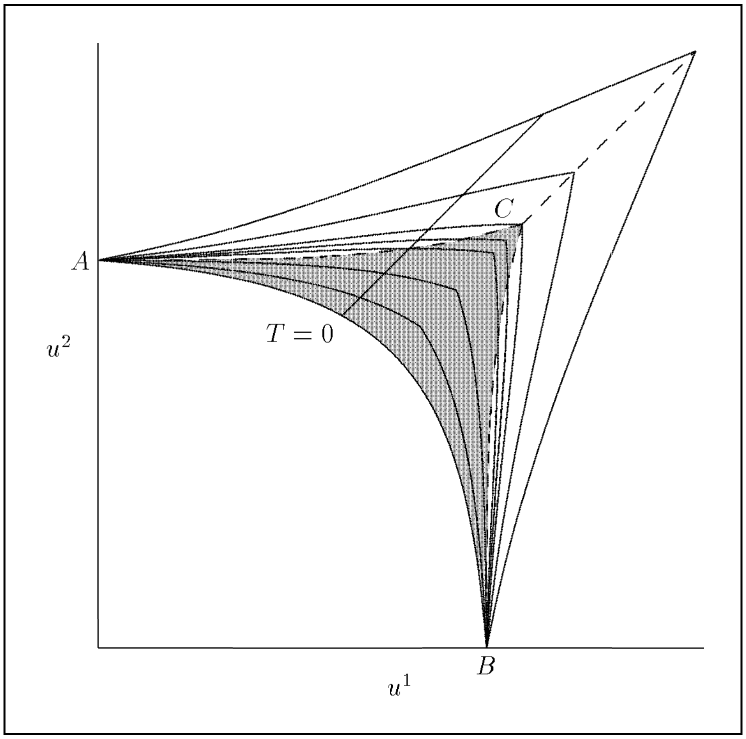

Figure 1: The effect of altruism (α)

because it reflects only the distribution of aggregate income $(w_1 + w_2)\bar{L}$. The points where $T = 0$, however, favour the high-wage individual, as shown in the figure. Due to the linearity of the altruism term, all points with a specific transfer T lie on a straight line.

The innermost curve represents the case without altruism ($\alpha = 0$). The altruism parameter α 'pushes' the utility frontier outward. The closer the consumption levels C^1 and C^2 are, the further out the utility possibilities move. Since labour supply is fixed, the locus where $C^1 = C^2$ coincides with $u^1 = u^2$. At these points the utility possibility frontier has a kink if $\alpha > 0$.

In the presence of altruism it is possible that not all points on a utility possibility frontier are Pareto optimal. In the example depicted in figure 1, an increase in the degree of altruism narrows the range of Pareto optima until, when $\alpha = \overline{\alpha}$, the set of Pareto optima is reduced to a single point on the utility possibility frontier. The shaded area indicates the range of Pareto optima for the different frontiers, and becomes smaller for larger α's. The points on the boundary of this area correspond to the voluntary transfer that an individual would choose ex post. One can thus identify the different ranges corresponding to proposition 5a. The intersection of the no-transfer line ($T = 0$) with the dashed boundary AC identifies $\underline{\alpha}$ and the corresponding utility possibility frontier. For lower values of α individual 2, despite being altruistic, would prefer negative T's, i.e. a transfer from the low-wage individual. Point C identifies $\overline{\alpha}$ where T becomes $\bar{T}_{max}$.

Determining the optimal transfer ex ante from

$$\max_T \frac{1}{2}u[w_1\bar{L} + T, \bar{L}] + \frac{1}{2}u[w_2\bar{L} - T, \bar{L}] + \alpha(w_1\bar{L} + T) + \lambda(w_2\bar{L} - T - w_1\bar{L} - T)$$

gives

Proposition 6

If the individuals are altruistic, labour supply is fixed and the transfer is decided on ex ante, then the unanimously agreed upon tax schedule will lead to identical consumption and utility levels for both individuals.

Altruism, thus, has no influence on the optimal tax (or private insurance)—if labour supply is fixed. However, in the presence of altruism it is possible that the ex post solution duplicates the ex ante one. This happens if there is a single Pareto optimum.

The problem becomes much more complicated if labour supply is variable. Ex post the individuals face a game in which both can provide the public good $C^1 = (w_1L^1 + T)$. Individual 2 can increase the minimum consumption by a transfer while individual 1 can increase his labour supply and subsequently his income and consumption to contribute to the public good. The Nash equilibrium in games of voluntary public-good provision generally has the property that the intended increase in provision by an individual's voluntary contribution is partially offset by the other players' reaction. In a Stackelberg solution the Stackelberg leader foresees this and therefore, typically, contributes less than under Nash behaviour.

The result in the present scenario is similar only because consumption and leisure are assumed to be normal goods. Individual 1 in his reaction to a larger transfer will then reduce his labour supply. This offsets the intended effect of the transfer, but only partially. Since consumption is normal too, the consumption of individual 1 will increase. For individual 2 the effect is of opposite sign: An increase in T will reduce C^2 und increase the labour supply L^2.

The individuals face the optimization problems (again taking account of the restriction $C^1 \leq C^2$):

$$\max_{L^2,T} u[w_2L^2 - T, L^2] + \alpha(w_1L^1 + T) + \lambda^2(w_2L^2 - T - w_1L^1 - T) \tag{13}$$

$$\max_{L^1} u[w_1L^1 + T, L^1] + \alpha(w_1L^1 + T) + \lambda^1(w_2L^2 - T - w_1L^1 - T)$$

As in the case with fixed labour supply, different levels of altruism give rise to different solutions. The necessary first-order conditions for an interior Nash equilibrium ($T > 0$, $C^1 < C^2$) are

$$u_C^2 = \alpha \tag{14}$$

$$-u_L^2/u_C^2 = w_2 \tag{15}$$

$$-u_L^1/u_C^1 = w_1(1 + \alpha/u_C^1). \tag{16}$$

They exhibit the recursive structure of the solution. The first two equations determine L^2 and T independently from individual 1's labour supply. I.e. for the question whether individual 2 would transfer some money, individual 1's income is relevant. However, it has no influence on the size of the transfer[4]. The third equation determines L^1 and this solution depends on a possible transfer. Note, the term α/u_C^1 does not reflect altruism in its literal meaning. It is positive and thus increases the right-hand side. If leisure is a normal good, then a higher α leads to a higher labour supply. The rationale is that the low-wage individual would like the minimum consumption in society to increase.

In a laissez-faire framework with ex ante private insurance and ex post voluntary transfers one encounters the Samaritan's dilemma (see Buchanan, 1975). In the presence of altruism,

[4]This issue is discussed and empirically investigated in Smith et al., 1995.

the high-wage individual may voluntarily transfer some money to the low-wage person. Anticipating this, there is an incentive to reduce private precautionary measures.

In the model of this paper, the individuals face this dilemma too. But contrary to other papers (cf. Bruce and Waldman, 1990, Coate, 1995 and the references cited therein) this leads to no efficiency loss.

Proposition 7

With private insurance and ex post voluntary transfers the individuals face the Samaritan's dilemma, but this does not lead to inefficiency.

Due to the linearity of the altruism term, the marginal benefit of a transfer is constant and independent of the poor person's insurance. An individual will, therefore, reduce private insurance by the full amount of the expected transfer. However, the optimality conditions (14)- (16) still hold.

Proposition 8

The voluntary transfer increases with the level of altruism.

Let

$$\Delta \equiv (w_2)^2 u^2_{CC} + 2w_2 u^2_{CL} + u^2_{LL}.$$

Then the Hessian matrix of (13) is

$$H = \begin{pmatrix} \Delta & -w_2 u^2_{CC} - u^2_{CL} \\ -w_2 u^2_{CC} - u^2_{CL} & u^2_{CC} \end{pmatrix}$$

and second-order sufficient conditions for a maximum are:

$$0 > \Delta$$

$$0 > u^2_{CC}$$

$$0 < u^2_{CC}\Delta - (w_2 u^2_{CC} + u^2_{CL})^2.$$

The effect of a change in α on T can be obtained by totally differentiating (14)-(15). This yields

$$dL^2 = dT\frac{w_2u^2_{CC} + u^2_{LC}}{\Delta}$$

$$d\alpha = (w_2u^2_{CC} + u^2_{CL})dL^2 - u^2_{CC}dT.$$

It follows that

$$\frac{dT}{d\alpha} = \frac{\Delta}{-u^2_{CC}\Delta + (w_2u^2_{CC} + u^2_{CL})^2} > 0.$$

Proposition 9

The interior Nash equilibrium in the game of ex post voluntary contributions is Pareto inefficient.

Maximization of the social welfare function, considering only Pareto optima with $C^1 \leq C^2$

$$\max_{L^1,L^2,T}(1-\gamma)u[w_1L^1+T, L^1] + \gamma u[w_2L^2 - T, L^2] + \alpha(w_1L^1+T) + \lambda(w_2L^2 - T - w_1L^1 - T)$$

leads to optimality conditions for an interior ($C^1 < C^2$) optimum:

$$u^2_C = \frac{\alpha}{\gamma} + \frac{1-\gamma}{\gamma}u^1_C$$

$$-u^2_L/u^2_C = w_2$$

$$-u^1_L/u^1_C = w_1(1 + \frac{1}{1-\gamma}\frac{\alpha}{u^1_C})$$

It is apparent that for $0 \leq \gamma \leq 1$ there is no social weight that duplicates the first-order conditions (14)-(16). It is worth noting that the outcome in the corresponding situation without altruism is Pareto optimal, i.e. altruism leads to inefficiency in this context.

If the Nash equilibrium occurs at a kink, the optimality conditions are different:

$$T = \frac{w_2L^2 - w_1L^1}{2}$$

$$\frac{-u^2_L}{u^2_C} = w_2(\frac{3}{2} - \frac{\alpha}{2u^2_C}) \leq w_2$$

$$u^2_C < \alpha$$

$$\frac{-u^1_L}{u^1_C} \geq w_1 \geq \frac{-u^1_L}{u^1_C + \alpha}.$$

Technically speaking, there are only two equations for three unknowns. The situation resembles Hirshleifer's (1983) 'weakest-link' construction which leads to a continuum of Nash equilibria. The situation is somewhat different here, because even at the kink where the consumption levels are identical, the situation is definitely asymmetric. If α is sufficiently large to support a kinked equilibrium, then individual 1 can and is willing to increase his consumption by working harder. Individual 2, on the other hand, can increase his consumption by decreasing the transfer.

Turning now to the *ex ante* determination of transfers we find that altruism complicates the problem by the public-good effect of minimum consumption. The optimization problem

$$\max_T \frac{1}{2}u[w_1L^1 + T, L^1] + \frac{1}{2}u[w_2L^2 - T, L^2] + \alpha(w_1L^1 + T) + \lambda(w_2L^2 - T - w_1L^1 - T),$$

where L^1 and L^2 solve the individual ex post optimization problems, gives rise to the following conditions

$$\begin{aligned} 0 &= w_1u_C^1 + u_L^1 + \alpha w_1 - \lambda^1 w_1 \\ 0 &= w_2u_C^2 + u_L^2 + \lambda^2 w_2 \\ 0 &= u_C^1 - u_C^2 + 2\alpha(w_1L_T^1 + 1) + 2\lambda(w_2L_T^2 - w_1L_T^1 - 2) \end{aligned} \tag{17}$$

where L_T^i denotes the effect of a change in transfers on the labour supply. If we restrict ourselves to an interior optimum, we can formulate

Proposition 10

If the individuals are altruistic, labour supply is variable and the transfer is decided on ex ante, then

a) *the transfers will be positive and*

b) *the resulting allocation is characterized by either*

- $C^1 = C^2$, $L^1 < L^2$ *and* $u^1 > u^2$, *or*
- $C^1 < C^2$, $L^1 < L^2$ *if* $u_{CL} > 0$ *or* $u_{CL} < 0$ *but 'small' in absolute value, or*

- $C^1 < C^2$, $L^1 > L^2$ *if* $u_{CL} < 0$ *and 'large' in absolute value*

In the absence of any transfers $u_C^2 < u_C^1$. An increase in T is therefore welfare increasing. One can rewrite (17) as

$$0 = u_C^1 - u_C^2 + 2\alpha C_T^1 + 2\lambda(C_T^2 - C_T^1).$$

Consider first the case where $C^1 < C^2$. Since consumption is normal, one can get the following inequalities

$$u_C[C^2, L^1] < u_C[C^1, L^1] < u_C[C^2, L^2] < u_C[C^1, L^2].$$

It follows immediately that $C^1 < C^2$ is inconsistent with separability, because in this case $u_{CL} = 0$ leads to $u[C^2] < u[C^2]$ which cannot hold.

The intuitive $L^2 > L^1$ would require $u_{CL} > 0$ as above. What about $u_{CL} < 0$? This would imply $L^2 < L^1$. It cannot be optimal for the ex ante choice of the labour supply for the same reason as in the preceding section. However, this argument may not hold under ex post choice of the labour supply. While it is true that $L^2 < L^1$ is Pareto inefficient, the Pareto superior allocation may not be reached with the labour supply chosen individually ex post.

The other set of possible solutions implies equal consumption $C^1 = C^2$, $L^1 < L^2$ and $u^1 > u^2$.

If the wage type is unobservable, we obtain the same results as in section 3. C_1 is *always* less than C^2. Therefore, altruism has no impact on the incentive constraint. We again have

$$u[C^2, \frac{Y^2}{w_2}] > u[C^1, \frac{Y^1}{w_1}],$$

the high-wage individual has to be better off.

5 Conclusion

In the absence of altruism, a transfer payment to the low-wage individual requires the government to guarantee the transfer or binding commitments to be possible. Not surprisingly, no voluntary transfers are made ex post. On the other hand, individuals agree on transfers ex ante since they are interested in insurance. These transfers can lead to 'over insurance' in the sense that the utility level of the recipient (of the transfer or insurance payment) can be higher than that of the donor. This situation cannot occur if transfers are not based on type but on income, and if the appropriate incentive constraint is taken into account.

The picture changes only slightly when individuals are altruistic. As one would expect, even for fixed labour supply the high-wage individual may voluntarily transfer income ex post. The amount depends on the strength of altruism. It is possible for the transfer to equalize income. The implications are essentially the same as above for ex ante decisions. The transfer is positive and can make the low-wage individual better off if there are no incentive contraints. Here, too, the transfer is positively correlated to altruism. But the resulting allocation is, in general, inefficent. Altruism seems to be an incomplete substitute for governmental redistribution.

References

Arrow, Kenneth J., 1981, Optimal and voluntary income distribution, in: Rosefielde, Steven S. (ed), *Economic welfare and the economics of Soviet capitalism—Essays in Honor of Abram Bergson*, Cambridge University Press, Cambridge, 267–288.

Becker, Gary S., 1981, Altruism in the family and selfishness in the market place, *Economica 48(189)*, February, 1–15.

Bruce, Neil and Waldman, Michael, 1990, The rotten-kid theorem meets the Samaritan's dilemma, *Quarterly Journal of Economics 105(1)*, February, 155–165.

Buchanan, James M., 1975, The Samaritan's dilemma, in: Phelps, Edmund S. (ed), *Altruism, morality and economc theory*, Russell Sage Foundation, New York, 71–85.

Coate, Stephen, 1995, Altruism, the Samaritan's dilemma, and government transfer policy, *American Economic Review 85(1)*, March, 46–57.

Hirshleifer, Jack, 1983, From weakest-link to best-shot: The voluntary provision of public goods, *Public Choice 41(3)*, 371–386.

Hochman, Harold M. and Rodgers, James D., 1969, Pareto optimal redistribution, *American Economic Review 59(4-Part I)*, September, 542–557.

Paqué, Karl-Heinz, 1986, *Philanthropie und Steuerpolitik—Eine ökonomische Analyse der Förderung privater Wohltätigkeit*, Mohr, Tübingen.

Smith, Vincent H., Kehoe, Michael R., and Cremer, Mary E., 1995, The private provision of public goods: Altruism and voluntary giving, *Journal of Public Economics 58(1)*, September, 375–394.

Statistisches Bundesamt, 1995, *Statistisches Jahrbuch 1995 für das Ausland*, Metzler-Poeschel, Stuttgart.

Stiglitz, Joseph E., 1987, Pareto efficient and optimal taxation and the new new welfare economics, in: Auerbach, Alan J. and Feldstein, Martin (eds), *Handbook of Public Economics II*, North-Holland, Amsterdam, 991–1042.

Summers, Lawrence H., 1989, Some simple economics of mandated benefits, *American Economic Review 79(2)*, May, 177–183.

Thurow, Lester C., 1971, The income distribution as a pure public good, *Quarterly Journal of Economics 85(2)*, May, 327–336.

Varian, Hal R., 1980, Redistributive justice as social insurance, *Journal of Public Economics 14(1)*, August, 49–68.