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Pigouvian taxes under imperfect competition if consumption depends on emissions

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1. Introduction

There is a considerable body of literature dealing with environmental regulation under imperfect competition. One typical result can be summarized as follows: The Pigouvian tax should be lower than marginal damage, whereas the tax should be equal to the marginal damage under perfect competition. There are few exception which will be mentioned below. The details depend on the technology, abatement possibilities, the demand side, and the market structure. The papers generally suppose that there are no repercussions of the external effect on the product market; i.e. demand and damages are independent or separable.

The last assumption is often unjustified and therefore dropped in the present note. A consumption good and an externality might be complements or substitutes. E.g. water quality influences the frequency of swimming in a lake or noise increases defensive expenditures. The paper considers a representative consumer whose utility depends on a commodity X and emissions E in a *nonseparable* way: The marginal willingness to pay for X is positively or negatively influenced by E. The good is supplied by a producer who is able to control the externality and therefore to influence the demand for his own product. It is reasonable to assume that the producer is not a price-taker. Therefore the case of monopoly and symmetric oligopoly will be examined.

The welfare analysis demonstrates that the formula for the optimal Pigouvian tax is more complicated than in the simple (separable) case. This is not surprising. But there are also some new non-expected insights:

First, even if the marginal damage in the optimal allocation equals zero, *two* instruments are needed to attain a first-best optimum. Second, even if the subsidy used to lessen the consequences of imperfect competition is low compared to marginal damage, a Pigouvian tax can be negative. The case can occur if X and E are complements. Third, the Pigouvian tax can be greater than marginal damage, even under imperfect competition. Substitutability of X and E is a necessary condition for this outcome. These results will be demonstrated for monopoly and afterwards extended to symmetric oligopoly.

Buchanan (1969) was the first author demonstrating the relevance of market structure. Smith (1976) and Barnett (1980) investigated the problem for monopoly in more depth. These papers showed that the optimal Pigouvian tax is lower than marginal damage or might even be negative. Ebert (1991) generalized the analysis to symmetric oligopoly with a fixed number of firms and various types of technology. Misiolek (1988) proved that the presence of rent seeking costs under monopoly can require the optimal tax rate to exceed the marginal damage. Katsoulacos/Xepapadeas (1995) demonstrated a similar result as Misiolek when the number of firms is endogeneous under oligopoly. It can also occur under uncertainty about the technology (Ebert (1996)). In all these papers consumption and pollution are independent.

The note is organized as follows: Section 2 presents the model and section 3 its solution for monopoly. In section 4 the findings are discussed and interpreted. Two cases have to be distinguished. Section 5 briefly extends the framework to symmetric oligopoly. Section 6 summarizes the results.

2. Model

There are three commodities, the consumption goods *X* and *Y*, and the externality *E*. A representative consumer possesses a quasi-linear utility function $U(X,E)+Y$. It is characterized by decreasing marginal utility of *X* $(U_X > 0 \quad \text{and} \quad U_{XX} < 0)$ and increasing marginal damages $(U_E < 0 \quad \text{and} \quad U_{EE} > 0)$, where U_X, U_{XX} etc. denote the respective partial derivatives. Utility maximization implies the inverse demand function for *X* $\quad p(X,E) = U_X(X,E)$, if *Y* is a numeraire good $(p_Y = 1)$. It is decreasing in *X* since $p_X = U_{XX}$. The cost function of *Y* is assumed to be linear (constant returns scale). The technology for *X* is characterized by the concave cost function $C(X,E)$ where marginal production cost is increasing $(C_X > 0, \quad C_{XX} > 0)$. *E* can increase or decrease costs $(C_E \gtrless 0)$, but marginal cost of *E* is increasing $(C_{EE} > 0)$. It is supposed that for every quantity *X* there is a level of emissions $E = E(X)$ such that $C_E(X, E(X)) = 0$ since otherwise corner solution have to be considered. $E(X)$ represents the cost-minimizing level of emissions. It increases (decreases) with *X* if the cross derivative C_{XE} is negative (positive). The marginal production cost C_X then decreases (increases) with *E*.

Now consider a profit-maximizing monopolist who produces *X* and pollutes the environment by *E*. Furthermore we assume that emissions are taxed by a unit tax *s* and consumption by a unit tax *t*. The maximization problem

$$\max_{X,E} \; p(X,E) \cdot X - C(X,E) - t \cdot X - s \cdot E$$

yields the first-order conditions

$$p_X(X,E) \cdot X + p(X,E) - C_X(X,E) - t = 0$$

$$p_E(X,E)\cdot X - C_E(X,E) - s = 0 \qquad (1)$$

which require that marginal revenue of *X* equals marginal production cost and marginal revenue of *E* equals marginal abatement cost, respectively. (Of course the respective tax has to be taken into account.) Here it has to be supposed that the optimization problem is well-defined and that its solution is unique. These assumption are made in the following; in particular it is assumed that *p* is twice continously differentiable or, equivalently, *U* is three times continously differentiable.

In order to describe the impact of taxation on the monopolist's behavior we perform a comparative statics exercise and obtain

$$\begin{pmatrix} dX \\ dE \end{pmatrix} = \frac{1}{\det H}\cdot\begin{pmatrix} p_{EE}\cdot X - C_{EE} & -(p_{XE}\cdot X + p_E - C_{XE}) \\ -(p_{EX}\cdot X + p_E - C_{XE}) & p_{XX}\cdot X + 2p_X - C_{XX} \end{pmatrix}\begin{pmatrix} dt \\ ds \end{pmatrix} \qquad (2)$$

where *H* denotes the Hessian which is negative definite $(\det H > 0)$. Therefore the coefficients of the main diagonal of *H* have to be negative

$$p_{EE}\cdot X - C_{EE} < 0 \quad \text{and}$$

$$p_{XX}\cdot X + 2p_X - C_{XX} < 0\,. \qquad (3)$$

That implies

$$dX/dt < 0 \quad \text{and} \quad dE/ds < 0\,.$$

The sign of $(p_{EX}\cdot X + p_E - C_{XE})$ is indeterminate as is, therefore, the sign of dX/ds and dE/dt.

3. Welfare analysis

We now turn to welfare analysis. The first-best allocation is derived by maximizing social welfare

$$W(X,E) := U(X,E) - C(X,E)\,.$$

The optimum is characterized by

$$U_X = C_X$$

and $\quad -U_E = -C_E,$

i.e. price has to equal marginal production cost and marginal damage has to be equal to marginal abatement cost. In view of the externality and monopoly the optimum will not be attained without governmental regulation. Then the monopolist's reaction to taxation has to be taken into account. We will consider three different cases:

(i) *s* and *t* are available

Employing the instruments one gets the necessary condition

$$(U_X - C_X)\frac{dX}{ds} + (U_E - C_E)\frac{dE}{ds} = 0$$

and $$(U_X - C_X)\frac{dX}{dt} + (U_E - C_E)\frac{dE}{dt} = 0 .$$

Elimination of C_X and C_E by means of the first-order conditions of profit maximization demonstrates that

$$s = -U_E + p_E \cdot X \tag{4}$$

and $$t = p_X \cdot X$$

are sufficient for a first-best optimum. The commodity tax corrects the implications of market structure: one has to subsidize the monopolist in order to reach a first-best market equilibrium. On the other hand the Pigouvian tax *s* has to take into consideration two effects of the externality: $-U_E$ corresponds to the (familiar) marginal damage. The second term reflects the marginal impact on the monopolist's revenue. The term will be discussed more deeply in the next section. It should be stressed that *s* may differ from zero even if the consumer suffers no damage at the margin $(-U_E = 0)$. That proves the first assertion stated in the introduction.

(ii) Using only *s*

By the same procedure we obtain the second-best solution

$$s = -U_E + p_X \cdot X \cdot \frac{dX / ds}{dE / ds} + p_E \cdot X . \tag{5}$$

In comparison to equation (4) a further term has to be taken into account. It represents a substitute for the unit tax *t* which is now unavailable. In principle $p_X \cdot X$ equals the amount by which commodity *X* has to be subsidized. Since that cannot be done directly in this case, emissions are subsidized instead. But the magnitude of the subsidy has to be recalculated or converted to the correct units by means of $\frac{dX}{dE} = \frac{dX/ds}{dE/ds}$.

The sign of the conversion factor is ambiguous:

$$\frac{dX/ds}{dE/ds} = \frac{-(p_{XE}X + p_E - C_{XE})}{p_{XX} \cdot X + 2p_X - C_{XX}}$$

since the sign of the numerator is indeterminate. The denominator is negative (equation (3)).

(iii) Using only *t*

In this case we get the analogue to equation (5):

$$t = -U_E \cdot \frac{dE/dt}{dX/dt} + p_X \cdot X + p_E \cdot X \cdot \frac{dE/dt}{dX/dt}, \qquad (6)$$

where

$$\frac{dE/dt}{dX/dt} = \frac{-(p_{EX} \cdot X + p_E - C_{XE})}{p_{EE} \cdot X - C_{EE}}. \qquad (7)$$

Again there are three effects which play a role: the consumer's and the monopolist's (marginal) damage (first and third term) and the subsidy (second term). Thus *t* must be a partial substitute for *s*. The corresponding terms also have to be converted appropriately. The sign of $\frac{dE/dt}{dX/dt}$ is indeterminate, too. This result is, in principle, already contained implicitly in Sheshinski's (1976) equation (4). He examined the regulation of monopoly in situations where demand for a consumption good is influenced by an index of quality. One of his several scenarios is formally equivalent to this case.

4. Discussion

The interesting and new part of the solutions derived above is the term $p_E \cdot X$. It requires some interpretation and motivation. The marginal change in the marginal willingness to pay (MWTP) p_E is given by the cross partial U_{XE} which may have either sign. It determines the relationship between *X* and *E*. *X* and *E* are complements if an increase in *E* decreases the MWTP for *X*, $p(X, E)$. This definition is equivalent to $p_E < 0$ or $U_{XE} < 0$. Similarly, *X* and *E* are substitutes if $p_E = U_{XE} > 0$. E.g. sun-shades and air pollution are complements. On the other hand window-panes and noise can be seen as substitutes.

Concentrating on the case in which only the Pigouvian tax *s* is available we have to explain the impact of $p_E \cdot X$. The term is negative if *X* and *E* are complements. For a moment the other terms in equation (5) are ignored. Then emissions are subsidized, which has intuitively the following consequences: emissions *E* are increased. Therefore damages increase as well (welfare loss ΔD), production costs decrease (gain ΔC), and demand and equilibrium quantity are reduced, since p_E is negative (loss in social welfare ΔW).

The interpretation is supported by a formal analysis. Let (X_s, E_s) and (X_0, E_0) denote the quantities in the tax and no-tax situation, respectively. Then the overall welfare gain is given by

$$\begin{aligned} &W(X_s, E_s) - W(X_0, E_0) \\ &= U(X_s, E_s) - C(X_s, E_s) - \big(U(X_s, E_S) - C(X_0, E_0)\big). \end{aligned}$$

This expression can be rearranged and decomposed to

$$\begin{aligned} &\Delta D + \Delta C + \Delta W = \left[U(X_s, E_s) - U(X_s, E_0)\right] + \left[C(X_s, E_0) - C(X_s, E_s)\right] \\ &+ \left[\big(U(X_s, E_0) - C(X_s, E_0)\big) - \big(U(X_0, E_0) - C(X_0, E_0)\big)\right] \end{aligned}$$

which reflect the welfare changes mentioned above. The first and third term are negative. They can be dominated by the second term, describing the cost-saving.

In the example one has to subsidize air-pollution. That reduces the purchase of sun-shades and costs. When the consumer buys fewer sun-shades her evaluation of air pollution changes. Air quality is no longer as important as before. Thus damages are reduced *indirectly*! Whenever we take consumer sovereignty serious we have to accept this result.

Up to now we neglected the other terms in equation (5). They are well-known. Marginal damage $(-U_E)$ is positive, the sign of the second term unclear. Therefore the sign of the total right hand side *can* also be positive.

Similarly, one can prove that the tax should exceed the marginal damage $(-U_E)$. Suppose that *X* and *E* are substitutes $(p_E \cdot X > 0)$. Investigation of the first-order condition of profit-maximization (equation (1)) proves that C_E has to be positive. The firm increases (!) emissions (compared to a cost-minimal solution) in order to increase its demand and profits. This case is hopefully of little relevance for environmental policy. Then taxation of *E* reduces emissions and damages. It also shifts the demand curve to the left and increases production costs. Therefore the consumption good is consumed less, implying an increase of the marginal damage $(\Delta(-U_E) = -U_{XE} \cdot \Delta X > 0$ since $U_{XE} > 0$ and $\Delta X < 0)$. If emissions are abated drastically, the gain may exceed the welfare loss. This *can* be accomplished by using a relatively high tax rate. Since the firm pollutes excessively the tax rate can be greater than marginal damage.

An analogous discussion applies to case (iii) when only a unit tax *t* is employed. The Appendix provides two numerical examples for $p_E < 0$ and $p_E > 0$, respectively.

5. Oligopoly

For symmetric oligopoly the result is essentially the same. Suppose that there are *n* (identical) firms, each producing the commodity *X* and having the technology described for the monopoly in section 2. Firms maximize profit and compete in quantities. Of course the market structure has to be taken into account. Let *X* and *E* be total output and total emission, respectively, of the industry. Then a first-best optimum is attained for

$$s = -U_E + p_E \cdot \frac{X}{n}.$$

and $$t = p_X \cdot \frac{X}{n}.$$

In principle the optimal tax rates can be interpreted as a generalization of those obtained for monopoly $(n = 1)$.

Looking at the literature this outcome is not surprising. The result is also obtained if only a Pigouvian tax *s* or an unit tax *t* is used. Then we get for the optimal rates

$$s = -U_E + p_X \cdot \frac{X}{n} \cdot \frac{dX/ds}{dE/ds} + p_E \cdot \frac{X}{n}$$

and $$t = -U_E \cdot \frac{dE/dt}{dX/dt} + p_X \cdot \frac{X}{n} + p_E \cdot \frac{X}{n} \frac{dE/dt}{dX/dt},$$

respectively. They depend on the number of firms belonging to the industry. If *n* is increased, the optimal tax rate *s* converges to the marginal damage $-U_E$ (and *t* to $-U_E \cdot \frac{dE/dt}{dX/dt}$). Then market structure and the indirect effect no longer play a role, because a single firm is a price-taker and is not able to influence the market price by its supply or emissions. But for oligopoly the conversion factor

$$\frac{dX/ds}{dE/ds} = \frac{-(p_{XE} \cdot X + n \cdot p_E - C_{XE})}{p_{XX} X + (n+1) p_X - C_{XX}}$$

is different from that derived above.

The other one $\left(\frac{dE/dt}{dX/dt}\right)$ is again given by equation (7). These factors are a direct consequence of the corresponding comparative statics, which generalizes equation (2):

$$\begin{pmatrix} dX \\ dE \end{pmatrix} = \frac{n}{\det H^*} \begin{pmatrix} p_{XE} \cdot X - C_{EE} & -(p_{XE} \cdot X + np_E - C_{XE}) \\ -(p_{EX} X + p_E - C_{XE}) & p_{XX} X + (n+1) p_X - C_{XX} \end{pmatrix} \begin{pmatrix} dt \\ ds \end{pmatrix},$$

where H^* denotes the matrix describing the total differential of the first-order conditions.

The interpretation is the same as in section 3 and 4. However the larger the number of firms the more relevant is the marginal damage $-U_E$.

6. Summary

The analysis demonstrates that repercussions of pollution to the demand side have to be taken into account, if firms are price-setter. As a consequence the optimal (Pigouvian) tax rate might be negative or even greater than marginal damage.

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Appendix: Examples

1) Define a quadratic utility function U by

$$U(X,E) := -a\frac{X^2}{2} + bX + \alpha \cdot X \cdot E - c\frac{E^2}{2} + d \cdot E \ .$$

Then we obtain

$$p(X,E) = U_X(X,E) = -aX + b + \alpha \cdot E \ .$$

The cost function is given by

$$C(C,E) = e\frac{X^2}{2} - fX + g\frac{E^2}{2} - hE \ .$$

The producer maximizes profit, taking into account the tax:

$$p(X,E) \cdot X - C(X,E) - s \cdot E \quad \rightarrow \max$$

or $$\Pi(X,E) = -a \cdot X^2 + b \cdot X + \alpha \cdot X \cdot E - (e\frac{X^2}{2} - fX + g\frac{E^2}{2} - hE) - s \cdot E \ .$$

The first-order conditions have to be fulfilled:

$$\frac{d\Pi}{dX} = (-2a \cdot X + b + \alpha E) - (eX - f) = 0$$

$$\frac{d\Pi}{dE} = \alpha \cdot X - (gE - h) - s = 0$$

They imply

$$X(s) = \frac{(b+f)g + \alpha(h-s)}{-\alpha^2 + g(2a+e)}$$

$$E(s) = \frac{\alpha(b+f) + (2a+e)(h-s)}{-\alpha^2 + g(2a+e)}.$$

Social welfare is defined by

$$SW(X,E) = U(X,E) - C(X,E).$$

Taking into consideration that $E = E(s)$ and $X = X(s)$ we maximize

$$SW\big(X(s), E(s)\big).$$

The first-order condition is

$$\frac{\partial SW}{\partial s} = \ldots \overset{!}{=} 0.$$

This complicated condition yields

$$s = \frac{\alpha^2(de - ah + 2ad) + \alpha(b+f)(2ac + ce - ag) + (ch - dg)(2a+e)^2}{-\alpha^2(3a+e) + (c+g)(2a+e)^2}$$

$$X = \frac{(2a+e)\big((b+f)(c+g) + \alpha(d+h)\big)}{-\alpha^2(3a+e) + (c+g)(2a+e)^2}$$

$$E = \frac{\alpha(b+f)(3a+e) + (d+h)(2a+e)^2}{-\alpha^2(3a+e) + (c+g)(2a+e)^2}$$

For an optimum it is required that

$$\frac{\partial^2 SW}{\partial s^2} = \frac{\alpha^2(3a+e) - (c+g)(2a+e)^2}{\big(-\alpha^2 + g(2a+e)\big)^2} < 0.$$

2) **<u>Example</u>** <u>1:</u> *X* and *E* are complements.

Using the model introduced above we set

$$a = 0.5, \quad b = 3, \quad c = 0.25, \quad d = 1$$

and $\quad \alpha = U_{XE} = -1.$

The cost function is determined by

$$e = 1, \quad f = 0, \quad g = 1, \quad \text{and} \quad h = 2\,.$$

Then the optimal Pigouvian tax is given by $s = -0.4$. It implies

$$X = 0.6, \quad E = 1.8, \quad \text{and} \quad U_E = -0.05.$$

3) **<u>Example</u>** <u>2</u>: *X* and *E* are substitutes.

Here $\alpha = U_{XE}$ has to be positive. For any **linear** utility function $(a = 0 \quad \text{and} \quad c = 0)$ and a cost-function satisfying $f = 0$, we obtain

$$s = -d$$

and $\quad U_E = \alpha X + d\,.$

Therefore

$$\begin{aligned} s &= -d = -(d + \alpha X) + \alpha X \\ &= -U_E + \alpha X; \end{aligned}$$

i.e. *s* exceeds the marginal damage $-U_E$ and is positive whenever *d* is negative.