

Wirtschaftswissenschaftliche Diskussionspapiere

**Exogenous preferences and endogenous tastes or
tastes as an object of preference***

Udo Ebert and Oskar von dem Hagen

V-177-97
November 1997

Abstract

A main result of this paper is that many different situations of variable tastes have a similar structure. Several so far completely different strands of economic literature can be analysed and discussed in the unifying framework presented.

Tastes are defined as orderings of a subspace of a (suitably chosen) commodity space. The analysis focuses on two main aspects of variable tastes: the institutional question of who influences or determines the individuals' tastes and the technological question concerning the properties of the cost function for taste changes. The allocative inefficiency of an economic system with endogenous tastes is demonstrated and Pigouvian-type corrective taxes are briefly discussed.

It is shown that changes in tastes have a direct and an indirect impact on welfare. The latter is due to changes in the equilibrium allocation. The literature on variable tastes often, sometimes mistakenly, ignores the direct effect, while the related theory of externalities focuses on it and disregards the indirect effect which disappears under customary separability assumptions.

KEYWORDS: variable or endogenous tastes/preferences, commodity space

JEL CLASSIFICATION: D11, D60

1 Introduction

“Almost every economist will agree that the assumption of fixed consumers’ preferences, though rather convenient, is a very unrealistic hypothesis” (Gaertner, 1974). This and similar statements from other authors seem to indicate that there is a fundamental flaw in traditional utility theory. However, we will argue in the following that the controversy of fixed vs. variable preferences is a red herring – or a question of semantics. Some confusion in this discussion arises from the non-standard use of standard terminology. Consumers are usually characterized by exogenous characteristics defined within an exogenous commodity space. The common terminology for the characteristics is ‘preferences’ and ‘endowment’ respectively. By definition, therefore, one cannot explain preferences within a given model. And without exogenous changes they remain fixed. Natural curiosity leads to the question: what happens if endogenous changes of preferences are allowed? At this point ambiguity sets in because there are then two ‘preferences’: the concept used before and a new meta-structure that determines it. This new meta-structure must again be exogenous because one cannot completely ‘endogenize’ the consumers’ preferences – this is a corollary of Gödel’s theorem (Gödel, 1931).

Some authors keep the term preference for the endogenized concept (e.g. Gintis, 1974, Day, 1986). This terminology is unfortunate because the term preference commonly refers to the exogenous characterization and it is easier to relate the extended theory to the standard version – and vice versa – if this convention is kept. Therefore, many authors use the term ‘taste’ for the endogenous concept (e.g. von Weizsäcker, 1971, Hammond, 1976). Harsanyi (1954) explicitly distinguishes ‘variable tastes’ and (fixed) preferences, even though he usually refers to the latter in the representation of an ‘extended ordinal utility function’.

Once tastes are defined, several positive and normative questions arise if changes in tastes are considered: what causes a change in tastes; who can influence or even control these causes; and, is it ethical for the government to influence individual tastes? What is the

*The authors wish to thank Richard Arnott and Heinz Welsch for helpful comments and Sylvélie Hartig for editing the English style.

adequate procedure for evaluating welfare changes if tastes are variable? The quest for general answers to these questions has led to a large body of literature. In contrast to the conceptual or philosophical approach of many authors the present paper focuses on pragmatic aspects of modelling variable tastes. Thus, our contribution is twofold: firstly, a suitable choice of the commodity space will allow a unified approach to several, so far largely unrelated, problems. Some of them are well known, some are unfamiliar. Within the framework proposed, private goods, public goods, quality, household production, even age and health can be regarded as taste variables or parameters. The unifying modelling allows one to apply the well-developed intuition of familiar problems to unfamiliar ones. Secondly, using this approach it is possible to answer the questions raised above.

The paper is organized as follows: the general discussion in section 2 presents the concept of variable tastes, its relation to irrational behaviour, and a suitably chosen commodity space. Tastes are then defined as an ordering over a subspace of the commodity space. A central aspect of variable tastes is the institutional question of who influences or even determines an individual's taste. Several categories are discussed where tastes are determined by present or past decisions of the consumer, by decisions of another rational agent, by equilibrium conditions or exogenously (deterministically or randomly). It will turn out that several of the examples of variable tastes are quite familiar to economists – even though under different headings.

Section 3 formalizes a special case where the variable tastes of the consumer are determined by another rational agent, the firm. The model also allows the analysis of the technology of producing taste changes and their welfare impact. It is shown that, in general, the standard welfare theorems do not hold anymore. The sources of the inefficiencies will be discussed. It will be argued that the government must respect the consumer's sovereignty with respect to preferences but not to tastes. From the many possible instruments available to improve social welfare, first-best and second-best commodity taxes are examined more closely in a comparative-statics framework. Section 4 summarizes the results.

2 Variable tastes

If some of the consumer's characteristics are variable, does that imply that a new micro-economic theory is needed? Will the two fundamental theorems of welfare economics still hold? Before examining these questions, even before defining the variable characteristic 'tastes' properly, it is helpful to discuss a simple example. In the context of this example it will be argued that variable tastes are just one possible view or explanation of observed behaviour.

Consider a static framework where a consumer buys commodities x and y in a market. Let observations of his budget and consumption bundle be depicted in figure 1 by the budget lines a and b and the corresponding choices A and B . Obviously, these observations are incompatible with the usual assumptions in economic theory.

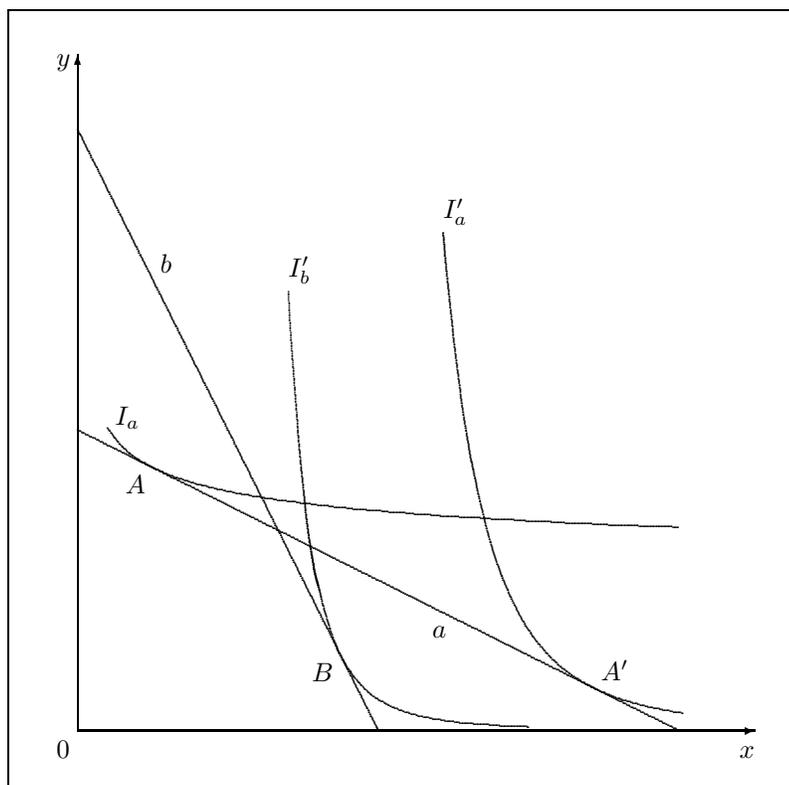


Figure 1: Irrationality or changing tastes

Facing these observations, a researcher could choose between three standard explanations:

1. the consumer is irrational,

2. his taste, assumed to be representable by taste indifference curves, has changed or
3. the consumer perceived differences in commodity bundles that the researcher did not notice.

The first explanation is unappealing. Nevertheless, several authors use observations like the ones depicted in figure 1 to ‘define’ irrational choice. See, e.g. Richter’s (1971) theorem 1 (“There exists an irrational choice”) and its proof.

Sen (1976), on the other hand, points out that irrationality (in his terms ‘inconsistency’) is just one possible explanation of the behaviour depicted in figure 1; another one is a change of tastes. Obviously, if the individual’s taste, represented by the indifference curve I_a , changes in such a way that the new taste can be represented by indifference curve I'_b , the individual chooses rationally in both cases. The catch of this argument is that ‘revealed taste changes’, as in figure 1, would render microeconomics almost useless, as it would be without any predictive power.

Both of these explanations differ from the third one in their assumption that the commodity space is given. However, one can observe the prices of goods and the quantities of goods bought, but one cannot observe ‘the commodity space’, i.e. the space of all commodities, in a very general sense, that are relevant for the consumer’s choice. Consider the well-known example of a person who never chooses the largest slice of cake offered but, say, always the second largest (see e.g. Sen, 1993). From three slices $\{a, b, c\}$ with sizes $a > b > c$ he would choose b while from $\{b, c\}$ he would choose c .¹ This is rational according to the choice rule stated and the same choice rule, representing the consumer’s taste, is applied in both cases. One way to resolve this dilemma is to assume that slice b from a choice set $\{b, c\}$ is not the same as slice b from a choice set $\{a, b, c\}$. If we denote the choice set by a subscript, the choice of c_{bc} from $\{b_{bc}, c_{bc}\}$ would be consistent with the choice of b_{abc} from $\{a_{abc}, b_{abc}, c_{abc}\}$.

In fact, many non-standard problems in economic analysis can be transformed into stan-

¹The problem of the standard theory with these choices is usually labelled ‘dependence on irrelevant alternatives’. The consumer seems to have revealed $c \succ b$. But in the presence of a , the alternative that is irrelevant in the sense that it is not chosen, he appears to reveal $b \succ c$.

dard models by a suitable choice of commodity space. Debreu (1976) showed that the standard general equilibrium model could be extended to uncertain or intertemporal environments by enlarging the commodity space with respect to time periods and states of the world, i.e. commodities with indices that refer to different time periods or different states of the world are regarded as different commodities. Arrow (1969) made externalities and public goods disappear by introducing quasi-private commodities.² In both of these cases the models with a greatly expanded commodity space provide important insights into the causes of ‘market failure’, namely that it can be explained as being caused by missing markets for contingent commodities, or external effects, etc. However, these models provide little insight into the problems of uncertainty, intertemporal utility maximization or public goods, i.e. they do not explain what happens if certain markets do not exist.

A similar situation arises in the context of variable tastes. One could enlarge the commodity space by indexing commodities with the actual taste. Then, by definition, tastes would not change anymore but ‘different tastes’ would be defined over different parts of the enlarged commodity space. In the context of figure 1 the taste indifference curves I' would not represent an ordering of x - y space but, say, an ordering of x' - y' space. This can explain market failure in the context of changing tastes but adds little to the understanding of an economy where markets for taste-contingent commodities do not form. On the other hand, these arguments indicate that there is no ‘natural’ definition of the commodity space. Instead, the proper choice of a commodity space may be crucial to the intuition that can be derived from a model.

The dimension of the commodity space is a decision on behalf of the analyst that involves a benefit and a cost. With a very small commodity space a model has a large predictive power – and a large potential of being rejected by data. With a larger commodity space a larger set of problems can be modelled (consistently). But the increase in the dimension of the commodity space involves a loss in predictive power. In the extreme, if all commodities were labelled by the exact time and date, all transactions would be rationalizable. But the predictive power of such a model would be zero because no two transactions would

²In Arrow’s model the externality for individual k caused by individual i ’s consumption of commodity j is a private good consumed by individual k only.

be the same.

In practice, therefore, the main body of literature on uncertainty and externalities does not use the large commodity spaces that Debreu (1976) and Arrow (1969) proposed. Instead, the standard procedure in these cases is to enlarge the commodity space by just one dimension, e.g. the externality or the state of the world.³ By analogy it seems advisable to keep the commodity space as small as possible in the context of endogenous tastes as well. While it may be necessary to use a vector of taste parameters for a specific problem, we will assume in the following that a single taste parameter is sufficient. Actually, the term taste ‘parameter’ is somewhat inaccurate because, depending on the actual problem, it may be a variable, a parameter or an exogenously fixed value.

Applying this idea to the theory of the consumer we introduce a taste parameter z and formally define the concept of taste:

Definition 2.1

*Let $u[x, y, z]$ represent the consumer’s preferences, where z is the taste parameter. Then we can define a **taste function** as*

$$T_z[x, y] \equiv u[x, y, z]. \quad (1)$$

It is important to note that the function u represents a fixed exogenous characteristic of the consumer. Even though u is an ordinal function and, therefore, not unique, it is apparent that admissible transformations of u must not depend on z . Furthermore, using the concept of a taste parameter, we are able to regard x and y as homogeneous commodities. If the term commodity space denotes the space over which preferences, represented by u , are defined then, formally, **tastes are an ordering of a subspace of the commodity space**.⁴ According to (1) the tastes can be represented in x - y space by taste indifference curves like I_a or I'_a in figure 1. Obviously, a change in the taste parameter z could explain why a rational consumer would choose A with budget a and tastes represented by I_a and would choose B with budget b and tastes I'_b .

³The prices of all commodities will generally vary with the externality or the state of the world – an issue that will be taken up in section 3.

⁴In (1) taste is defined by keeping z fixed. However, it is also possible to define taste functions $T_x[y, z]$ or $T_y[x, z]$.

The institutional setting

The definitions of commodity space, preferences and tastes are independent of the institutional framework. Furthermore, the taste parameter z is, presently, just an abstract concept without content. The economic significance of variable tastes, however, crucially depends on the institutional question of who determines the taste parameter z ? In discussing this question we will also present examples that are typically related to a specific institutional setting. We will discuss five institutional categories, where z is determined

1. by present decisions of the consumer,
2. by past decisions of the consumer,
3. by another rational economic agent,
4. by market or game-theoretic equilibrium conditions or
5. exogenously, but may change over time in a deterministic or random fashion.

We will discuss these categories in turn.

1. If z is a private good which the consumer can buy in the market, the concept of taste is formally well defined, well understood – and of little use. The interpretation of x - y taste indifference curves for a given value of z is straightforward but, generally, has little analytical content. The consumer simultaneously determines x , y and z and the optimal choice of z is, in general, a function of all prices and exogenous income. A change in the price of y will affect the choice of z and thus generate a new x - y taste indifference map.

An exception to this are quasi-linear preferences, i.e. preferences that can be represented by a utility function which is linear in z . Consider a consumer with utility function $u[x, y, z] = v[x, y] + z$ and endowment \bar{z} . Let $\{\hat{x}, \hat{y}, \hat{z}\}$ denote the optimal choice at given prices and consider the effect of a cut in endowment by $\Delta > 0$. Due to the structure of preferences the income effect is zero for goods x and y . The expenditure for these goods remains unchanged as do the shape of the x - y taste indifference curves and the point of

tangency. The new optimal commodity bundle is then $\{\hat{x}, \hat{y}, z'\}$ with $z' < \hat{z}$.⁵ Obviously, the consumer is worse off under the new endowment. Formulated in terms of the x - y taste functions:

$$T_{z'}[\hat{x}, \hat{y}] < T_{\hat{z}}[\hat{x}, \hat{y}].$$

Although the taste indifference curves corresponding to $T_{z'}[x, y]$ and $T_{\hat{z}}[x, y]$ coincide in x - y space, they represent different utility levels.

The expression ‘different utility levels’ may warrant a more detailed discussion because utility functions are an ordinal concept. However, different taste functions of a consumer are related. Harsanyi (1954) pointed this out at an early date but his argument was often ignored in the subsequent literature. He argued that a change in tastes has two aspects. The first aspect concerns the shape of the taste indifference curves. He called the second aspect ‘correspondence rule’ because this rule relates indifference curves (for different tastes) that correspond to the same utility level. Even though the utility level has no meaningful economic interpretation the expression ‘correspond to the same utility level’ does. Boadway and Bruce (1984) call this relation ‘ordinal scale full comparability (OS-FC)’. In the present context of variable tastes this implies that admissible transformations of the utility function u are also admissible transformations of taste functions T . However, all relevant taste functions must be subjected to the same transformation.

The discussion, so far, was just a rephrasing of standard analysis. A slight variation, however, bears out the complications and problems researchers face in situations of taste changes. Kapteyn (1994) discusses the case where the good z is not bought in a market but provided by household production. He points out that the observation of tastes (‘demand data’) is insufficient to derive a household cost function or to recover the consumer’s preferences.

⁵Assuming an interior optimum, i.e. $p_z(\bar{z} - \Delta) \geq p_x\hat{x} + p_y\hat{y}$, the following must hold

$$z' = \bar{z} - \Delta - \frac{p_x\hat{x} - p_y\hat{y}}{p_z} < \bar{z} - \frac{p_x\hat{x} - p_y\hat{y}}{p_z} = \hat{z}.$$

2. A large body of literature related to variable tastes deals with the problems of habit formation, addiction or other situations where past decisions determine present tastes.⁶ Common examples are alcohol, cigarettes or other drugs where z would represent past (possibly aggregate) consumption. In other cases z could be interpreted as human capital acquired by education (Gintis, 1974) or real capital accumulated by purchases of durable goods.

This is closely related to the puzzling subject of preferences over tastes (usually called ‘preferences over preferences’ in the literature). Bertrand Russell (see Gintis, 1974, p.419) noticed that we may prefer bridge to poetry but think it is better to prefer poetry to bridge. These ‘second-order’ preferences (Jeffrey, 1974) raise several questions that lie outside our focus on endogenous tastes. The basic argument in many of these cases is that past decisions, especially consumption decisions, lead to the present tastes. A ‘regret’ of past consumption decisions cannot be caused by changing tastes (alone), but only by an incorrect anticipation of these changes.⁷

3. The situation is different if another agent – another consumer, a firm or the government – determines the taste parameter z .⁸ Other consumers may consume goods with negative or positive external effects. Here, z could represent cigarettes, immunization, education etc. The other consumer generally ignores these effects. On the other hand, a firm may take account of these effects to increase its profits. If z is quality, a firm, obviously, will increase the quality of a product only if the willingness to pay increases with quality.⁹ Formally closely related to the provision of quality is advertising, at least in its function of increasing the willingness to pay for certain goods.¹⁰ Pollution is another example where

⁶See e.g. Gorman (1967), von Weizsäcker (1971), Hammond (1976), Stigler and Becker (1977), Yaari (1978) and Becker (1996) for further references. Several of these articles focus on a myopic consumer who maximizes his taste function instead of (long-run) utility, a question that lies outside the scope of the present paper.

⁷Without uncertainty and with perfect foresight, ex-ante and ex-post optimization must lead to the same result, i.e. the same optimal decisions. In such a context it makes perfect sense to optimize the eating of an appetite-arousing cake (Yaari, nd).

⁸In a sense, though, it is similar to the preceding case because past decisions of a myopic consumer can be regarded as decisions by somebody else. After all, a drunkard was not a drunkard when he started drinking.

⁹Optimal and equilibrium quality are an important topic in industrial organization. See, e.g. Dorfman and Steiner (1954), Levhari and Peles (1973) and Sheshinski (1976).

¹⁰See, e.g., Dorfman and Steiner (1954), Galbraith (1958), Dixit and Norman (1978), and Schmalensee (1986).

firms influence the willingness to pay. A firm that sells bathing suits may clean its waste water because it knows that polluted lakes reduce the willingness to pay for swimsuits. On the other hand, a firm that sells water filters has an interest to pollute water because this would allow higher prices for filters.¹¹ Finally, the government could produce z by providing public or private goods to maximize social welfare.

4. All multi-agent externalities belong in this category, i.e. externalities that are produced by several firms or several consumers. Examples are the voluntary provision of public goods as well as voting on public goods. In these situations several rational agents contribute to the determination of the taste parameter by individual calculation. However, the outcome of these activities may not be individually or socially optimal. Such a situation may, e.g., occur if the number of smokers or tax evaders in an economy is regarded as taste parameters. A related situation arises if consumption is influenced by the desire to stand out – or not to, as discussed in the classic contributions of Veblen (1899), Duesenberry (1949) and Leibenstein (1950). This “social character of individual consumption patterns” (Gaertner, 1974) can be regarded as a taste parameter that belongs to this group.

5. This category covers a wide variety of economic models, several of which have been mentioned before. E.g. z can represent a state of the world like rain or sunshine, a time period or season, individual characteristics like a time preference rate, age, sex or health.¹² In a static certain environment these cases are uninteresting: the tastes are given and do not change. The willingness to pay for rain, age or time, e.g., is well defined (in those three cases it is probably negative), but of little relevance. In an uncertain or intertemporal environment this is a very interesting category to be examined. The fact that the taste parameter is determined exogenously does not make the willingness to pay for it an irrelevant concept. Even though a consumer cannot buy a younger age or a lower time-preference rate, these taste parameters – through the willingness to pay for them – have an important influence on intertemporal consumption decisions. The

¹¹See Farzin (1996) and Ebert and von dem Hagen (1997) for more examples and further references.

¹²Health is probably determined, to a certain extent, by individual behaviour and for some individuals it has been possible to change their sex. Nevertheless, the exogenous aspects of these parameters dominate and therefore these cases should belong to this category.

time-preference rate, interpreted as taste parameter, is an excellent example.

It should be emphasized that a crucial aspect of endogenous tastes as formulated above is that the ‘cause’ z of tastes is a subject of the individual’s preference. This does not contradict Broome (1993),¹³ but implies that a situation where the consumer has no preferences with respect to his tastes is of little interest. In fact, if the individual’s preferences have the property that $u_z \equiv 0$, then the x - y taste indifference curves are unaffected by any change in z .

In most examples given above it is easy to see whether the consumer prefers a higher or lower value of the taste parameter. But reconsider the addictive influence of alcohol. If z is the past (aggregate) consumption of alcohol and y is today’s consumption, then it is generally agreed that the higher past consumption z increases the willing to pay for today’s consumption y . However, to derive any statements about welfare effects it is, loosely speaking, of vital importance whether an individual with an increased past consumption derives ‘more utility’ from alcohol or ‘less utility’ from the other good(s). In other words:

$$T_z^1 = zf[y] + g[x] \quad \text{and} \quad T_z^2 = f[y] + g[x]/z$$

may represent the same taste indifference curves between x and y but definitely not the same preferences. Thus, it is inadequate, to use taste indifference curves to evaluate welfare changes.

A related issue is addressed by Pollak and Wales (1979) who consider the family size as a taste parameter. In their terminology tastes are called ‘conditional preferences’ as compared to ‘unconditional’ preferences. In this case, as in the previous example, the taste parameter is given by past decisions. Formulated in terms of tastes vs. preferences it is natural to accept Pollak and Wales’ argument that preferences, and not tastes, are the relevant concept for welfare analysis.

¹³ “When I say ‘a cause of preference is not an object of preference’, I do not mean that no cause of preference is ever an object of preference” (Broome, 1993).

At this point it is worthwhile to reiterate Harsanyi's (1954) argument: a taste change has a direct and an indirect effect on welfare. The direct effect changes the welfare of any given allocation, while the indirect effect comes about if (and only if) the allocation changes, i.e. if the consumer chooses a different commodity bundle because the shape of his taste indifference curve has changed. The total influence on welfare must, of course, account for both effects.¹⁴

3 Welfare and variable tastes

The general discussion showed the large variety of effects that can be interpreted as being caused by changes in tastes. Furthermore, the crucial role of the institutional framework was demonstrated. However, to analyze the welfare implications of variable tastes, another important aspect has to be examined: the technology of producing taste changes. To focus the subsequent analysis on these two issues, we will restrict ourselves to a specific institutional setting: a single firm controls the taste parameter of a single consumer (this corresponds to category 3 in the preceding section). It is tempting to use the example given there that is most closely related to discussions about variable tastes, namely advertising.

However, we will choose a more abstract setting for several reasons: Firstly, there are several aspects of advertising, like information or commitment, that would blur the issue of changing tastes. Secondly, its production technology is too specific in the sense that advertising is (almost¹⁵) always costly. In general, taste changes may be the result of reductions in production costs. Finally, the aim of this paper is to present a unifying framework for variable tastes and to show that its basic structure is well known in economics.

We will, therefore, choose an interpretation that closely resembles a general externality,¹⁶

¹⁴Kaufman's (1995) ignorance of this argument is illuminating. He formally takes account of the direct effect but ignores it in his interpretation. With consumer preferences that correspond, in our notation, to $u = zf[y] + g[x]$ he examines whether the "adoption" of a higher z increases welfare. Not surprisingly it does, because of the positive direct effect. However, his example does illustrate that a positive direct effect may be dominated by the indirect one in an economy with several individuals. I.e. if a single individual "adopts" a higher value of z , he may end up worse off.

¹⁵'Product placement' could be regarded as an exception.

¹⁶There are some formal analogies to Ebert and von dem Hagen (1997), the optimal tax rates especially

the difference being that the taste change may not be a by-product of the firm's activity but may be its specific target. To simplify the analysis we will restrict ourselves to a static equilibrium framework with quasi-linear preferences of the consumer. The model is designed to illustrate the types of inefficiency that can arise and the use of government tax instruments to correct them either completely or partially. First, the social optimum will be discussed. This will afterwards be compared to the market solutions with a competitive and a monopolistic firm respectively. Finally, first-best and second-best commodity taxation by the government is derived.

Consider an economy with a three-dimensional commodity space. y and x are private consumption goods. The third good z assumes the role of the taste parameter. x will serve as numéraire. It may be thought of as leisure and it is assumed that the individual is endowed with a fixed amount \bar{x} of x only. To avoid unnecessary complications due to income effects we assume that tastes are linear in x . Furthermore, let all functions be sufficiently differentiable. The consumer's preferences can then be described by a utility function

$$U = u[y, z] + x. \quad (2)$$

It is assumed that the marginal utility of y is positive¹⁷ while that of z may be positive or negative and that z is bounded by $0 \leq z \leq \bar{z}$. A firm produces y and z with input x . Let $c[y, z]$ denote the private (and social) production costs measured in terms of the numéraire commodity x . An important aspect of the cost function is that it may be increasing or decreasing in z .

The social optimum can be determined by maximizing the utility of the consumer (2) subject to the resource constraint

$$\bar{x} \geq x + c[y, z]$$

are almost identical. However, the objective of that paper is completely different and it contains no reference to endogenous tastes at all.

¹⁷The marginal utility of y is an ordinal concept within the given preferences since it represents the marginal rate of substitution between y and x : $u_y = U_y/U_x$.

and restrictions on z .¹⁸ For an interior optimum it is necessary that

$$u_y = c_y$$

$$u_z = c_z.$$

An important property of the latter equation is that u_z and c_z may be positive or negative. Obviously, it is a necessary (but not sufficient) condition for an interior social optimum that the marginal utility of the taste parameter z and its marginal cost be of the same sign. Either z is a ‘good’ for consumers that is costly to produce, or it is a ‘bad’ for consumers but reduces production costs. A typical example of the latter case would be harmful emissions of a firm as a by-product of the production of y . If the marginal benefit and the marginal cost are of opposite sign or if the second-order condition is violated, a boundary solution must be optimal. See the appendix for the exact conditions under which boundary optima are optimal.

Market economy

x will serve as numéraire with a price of unity. It is assumed that the taste parameter z is not a marketable commodity. This is equivalent to a price of zero. Only the price of y is variable and will be denoted by p . These prices are consumer prices. They will differ from the producer prices if the government levies unit taxes τ_y and/or τ_z on y and z respectively.

The consumer aims to maximize his utility (2) subject to the budget constraint

$$\bar{x} \geq x + py.$$

An interior optimum requires

$$p = u_y. \tag{3}$$

The revenue of a price-taking firm is then py and such a firm maximizes its profit

$$\pi = py - c[y, z] - \tau_y y - \tau_z z \tag{4}$$

¹⁸Additional restrictions are the non-negativity constraints for x and y . They are formally acknowledged in the appendix.

subject to the upper limit on z . An interior optimum requires¹⁹

$$p = c_y + \tau_y \tag{5}$$

$$0 = c_z + \tau_z. \tag{6}$$

Without taxes, a competitive firm will choose an interior value of z ($0 < z < \bar{z}$) only if its provision reduces production costs at lower values and raises them at higher values of z . Obviously, the competitive firm ignores any influence on the consumer through the change of tastes. It is then easy to see that the two fundamental theorems of welfare economics are invalid in this context whenever $u_z \neq 0$.²⁰ In this case a single tax $\tau_z = -u_z$ (and $\tau_y = 0$) implements the first-best optimum (under suitable second-order conditions).

If the firm is the monopolistic supplier of y and z , its profit is still given by (4). But this firm takes account of the demand function defined by (3). Costs and other constraints are identical to those of the competitive firm. The conditions for an interior optimum now become

$$u_{yy}y + u_y = c_y + \tau_y \tag{7}$$

$$u_{yz}y = c_z + \tau_z. \tag{8}$$

For the second-order conditions involving third-order derivatives of the utility function that are commonly associated with monopoly problems, see the appendix.

The monopolistic firm takes account of its influence on the consumer's tastes but only for selfish reasons that may be unrelated or even detrimental to welfare. In a market equilibrium without taxes the firm takes account of the fact that an increase in the supply of y will lower its price ($u_{yy} < 0$). This is well known. But it will also consider the effect of the non-market good z on the price of or willingness-to-pay for y (u_{yz}). While the relevant social benefit of the taste parameter z is the direct effect u_z , the firm is interested only in the indirect effect u_{yz} on the price of y . Obviously, both effects could be of opposite sign. Provision of z , e.g. education, may have a direct benefit but certainly

¹⁹For the second-order conditions and boundary solution see the appendix.

²⁰Even if $u_z = 0$ at the social optimum, i.e. the socially optimal values of y and z would satisfy the firm's first-order condition, the market equilibrium may be an inefficient boundary solution. This could be due to the different second-order conditions (see appendix).

lowers the willingness-to-pay for certain goods. A monopolistic provider of such goods would, therefore, lower the provision of z compared to a competitive firm that ignores any influence on the consumer. The social planner, on the other hand, would increase the provision because the competitive firm ignores the beneficial effect of z .

As an example, consider a firm that produces water filters and, as a joint product, waste water that pollutes a public water reservoir. The competitive firm would produce the cost-minimizing amount of waste water. The social planner would prefer less pollution. Since the firm is indifferent between a small increase or decrease in z since the consumer has a positive willingness-to-pay for cleaner water, an end-of-pipe cleaning of the waste water would increase social welfare. The monopolistic firm, on the other hand, would recognize that dirtier water would increase the price of water filters. It would, therefore, increase the production of waste water – even though this raises production costs!

Corrective taxes

Hopefully, there will be no disagreement that a government should be allowed to take corrective measures against such behaviour even if this requires the government to disregard consumer's sovereignty with respect to tastes. The optimal choice of tax instruments allows a first-best solution in the monopoly case as well. However, two instruments τ_y and τ_z are necessary to remedy the two distortions.

If only one instrument, say τ_z , is available, the government may be unable to reach a first-best solution. The welfare optimization problem becomes

$$\max_{\tau_z} u[y[\tau_z], z[\tau_z]] - c[y[\tau_z], z[\tau_z]],$$

where $y[\tau_z]$ and $z[\tau_z]$ are defined, implicitly, by the first-order conditions for an interior optimum of a price-taking (5)-(6) or a monopolistic firm (7)-(8), respectively, and by $\tau_y = 0$.

The necessary first-order condition for an interior social optimum

$$(u_y - c_y) \frac{\partial y}{\partial \tau_z} + (u_z - c_z) \frac{\partial z}{\partial \tau_z} = 0 \tag{9}$$

involves the reactions by the firms. If the firm is a price-taker, it follows from (3) and (5) that $u_y = c_y$. (6) can be used to substitute τ_z for $-c_z$. (9) then becomes

$$\tau_z = -u_z$$

which was derived earlier by inspection of the first-order conditions. In this case the single instrument allows the implementation of a first-best optimum.

The second-best tax with monopolistic distortions is more complicated. Using (8) to substitute for c_z one obtains

$$\tau_z = -u_z + u_{yz}y + (u_y - c_y)\frac{dy}{dz}.$$

dy/dz can be derived by totally differentiating the firm's first-order condition for z . The optimal tax then becomes

$$\tau_z = -u_z + u_{yz}y + (u_y - c_y)\frac{u_{yyz}y + u_{yz} - c_{yz}}{u_{yyy}y + 2u_{yy} - c_{yy}}.$$

The monopolist ignores u_z and inefficiently, from a social point of view, accounts for the effect of z on p . In addition to these distortions in the z sector the optimal tax also has to take account of the monopolistic distortion in the y market that leads to $u_y > c_y$. The distortion has to be evaluated in terms of z instead of y and is, therefore, multiplied by a conversion factor which corresponds to dy/dz .

If only y can be taxed, i.e. $\tau_z = 0$, the formulae for τ_y can be derived in a similar fashion. Since the competitive firm takes the price as given, it reacts only to changes in costs. It is, therefore, not surprising that in this case the indirect intervention in the z sector through τ_y is optimal only if $c_{yz} \neq 0$. The optimal tax can be derived as

$$\tau_y = u_z \frac{c_{yz}}{c_{zz}}.$$

The tax for a monopolistic firm includes an additional term that reflects the direct influence of the tax on the monopoly price p . The optimal tax is

$$\tau_y = u_{yy}y + (u_z - c_z)\frac{u_{yyz}y + u_{yz} - c_{yz}}{u_{yzz}y - c_{zz}}.$$

The model of this section was intentionally similar to the externality problem so that well-developed intuition can be applied to the unfamiliar problem of variable tastes. Thus, we were able to

- show that variable tastes may invalidate both fundamental theorems of welfare economics and that the government may and should consider correcting possible market failures.
- demonstrate that, with a suitably chosen commodity space, the standard theories can be applied to the problem of variable tastes.
- emphasize that the relevant foundation to measure welfare changes in the presence of variable tastes are not the tastes but the underlying preferences. Hopefully, this will contribute to the resolution of a controversial issue.

To a certain degree the approach of this section is susceptible to the criticism of ‘Nirvana economics’ because the government is assumed to know the preferences and tastes of a consumer. However, it appears to be generally accepted that the government, compared to the individual, has a less subjective view of statistical data concerning, e.g., safety measures or addictive effects of consumption goods. Individuals often falsely believe that they are less prone to accidents or addiction than the average person. In this sense, the government may know the influence of certain taste parameters better than the individual.

4 Conclusion

It is demonstrated in the paper that variable tastes are, in principle, well known in economic theory. If tastes are governed by an additional variable, there is, *prima facie*, no difference compared to other goods in the economy. If there are more than two consumption goods, the indifference curves between any two of them are usually shifted if the consumption of other goods varies. In the terminology of this paper these shifts can be interpreted as changes in taste.

One might argue that one difference is that the taste parameter z is often not controlled by the consumer. But this is quite common in economic theory: public goods, external effects or the quality of goods are not, usually, controlled by the consumer except possibly to a small degree. However, this does not imply that these ‘parameters’ as viewed by the

consumer do not affect his well-being. On the contrary: the marginal utility or disutility of these ‘parameters’ are the core element in explaining market failures. In this sense some of the cases discussed in the present paper tell only different stories about known facts.

However, it is interesting to note that the main body of literature on, say, externalities focuses exclusively on the direct effect of the taste parameter (u_z in the present paper) and assumes that there is no indirect effect (equivalent to $u_{yz} \equiv 0$). On the other hand, the literature on changing tastes often focuses on the indirect effect that is related to the shape of the taste indifference curves and, in this case mistakenly, ignores the direct effect: The taste parameter z affects the shape of the x - y taste indifference curves only if $u_{yz} \neq 0$, and this is incompatible with the assumption that $u_z \equiv 0$. The direct influence of the taste parameter on welfare must not be ignored.

The formal modelling in section 3 was used to show the inefficiencies of a market equilibrium, the importance of the technology (cost function) of producing changes in tastes and the optimal use of commodity taxation that takes account of taste changes. The results are familiar to a certain degree. Optimal taxes include terms that reflect the direct and the indirect influence of the taste parameter. The second-best taxes involve corrections for distortions in the other sector.

Since we restricted ourselves to a comparative static analysis, there is no theory of taste formation in this paper. But the comparative advantage of economists is to analyse positive and normative aspects of given (meta-)preferences – not to explain or derive preferences, tastes or their changes.

One of the issues that was touched only briefly concerns the foresight of an individual with respect to preference changes. Does a student know how much his studies will ‘bend’ his indifference curves or did an addict foresee his addiction? Probably not. The future always involves some uncertainty. However, the consequence cannot be to disregard these effects completely. Instead, one should assume that the consumer has some subjective expectations as to the changes in his tastes. This is also true for the other economic agents, firms and government, who try to influence the taste of a consumer.

As regards the question whether the government should leave tastes in the realm of consumer sovereignty, the answer should be a clear no. Nevertheless, the question which tastes should be influenced, in which way and to what extent is a delicate ethical issue – especially if the consumers incorrectly anticipate the effects of taste parameters e.g. by ignoring the addictive effect of drugs. These cases are closely related to the literature on merit goods that began with Musgrave (1959). The unfortunate aspect from a the viewpoint of political economy is that individuals who have wrong expectations about the effect of the consumption of certain goods on tastes may never learn that the government’s corrective measures are beneficial.

Despite the many different situations of variable tastes, a main result of this paper is that they all have a similar structure. Many so far completely different strands of economic literature can be analysed and discussed in the unifying framework presented. Hopefully, this will stimulate further research.

References

- Arrow, Kenneth J., 1969, The organization of economic activity: Issues pertinent to the choice of market vs. non-market allocation, in: *The analysis and evaluation of public expenditures: The PPB system 1*, U.S. Government Printing Office, Washington D.C., 47–64. 5, 6
- Becker, Gary S., 1996, *Accounting for tastes*, Harvard University Press, Cambridge. 9
- Boadway, Robin W. and Bruce, Neil, 1984, *Welfare economics*, Blackwell, Oxford. 8
- Broome, John, 1993, A cause of preference is not an object of preference, *Social Choice and Welfare* 10(1), January, 57–68. 11, 11
- Day, Richard H., 1986, On endogenous preferences and adaptive economizing, in: Day, Richard H. and Eliasson, Gunnar (eds), *The dynamics of market economies*, North-Holland, Amsterdam, 153–170. 1
- Debreu, Gerard, 1976, *Theory of value - An axiomatic analysis of economic equilibrium*, Yale University Press, New Haven. 5, 6
- Dixit, Avinash K. and Norman, V., 1978, Advertising and welfare, *Bell Journal of Economics* 9(x), January, 1–17. 9
- Dorfman, Robert and Steiner, Peter O., 1954, Optimal advertising and optimal quality, *American Economic Review* 44(5), December, 826–836. 9, 9

- Ebert/von dem Hagen: Exogenous preferences and endogenous tastes 21
- Duesenberry, James S., 1949, *Income, saving, and the theory of consumer behavior*, Harvard University Press, Cambridge. 10
- Ebert, Udo and von dem Hagen, Oskar, 1997, Pigouvian taxes under imperfect competition if consumption depends on emissions, *Wirtschaftswissenschaftliche Diskussionsbeiträge V-170-97*, Universität Oldenburg. 10, 12
- Farzin, Y. Hossein, 1996, Can higher environmental standards enhance competition and welfare?, *Nota di Lavoro 87.96*, Fondazione Eni Enrico Mattei, Milano. 10
- Gaertner, Wulf, 1974, A dynamic model of interdependent consumer behavior, *Zeitschrift für Nationalökonomie 34(3-4)*, 327–344. 1, 10
- Galbraith, John Kenneth, 1958, *The affluent society*, Houghton Mifflin. 9
- Gintis, Herbert, 1974, Welfare criteria with endogenous preferences: The economics of education, *International Economic Review 15(2)*, June, 415–430. 1, 9, 9
- Gödel, Kurt, 1931, Über formal unentscheidbare Sätze der Principia Mathematica und verwandter System, I, *Monatshefte für Mathematik und Physik 38*, 173–198. 1
- Gorman, W.M., 1967, Tastes, habits and choices, *International Economic Review 8(2)*, June, 218–222. 9
- Hammond, Peter J., 1976, Endogenous tastes and stable long-run choice, *Journal of Economic Theory 13(2)*, October, 329–340. 1, 9
- Harsanyi, John C., 1954, Welfare economics of variable tastes, *Review of Economic Studies 21*, 204–213. 1, 8, 12
- Jeffrey, Richard C., 1974, Preference among preferences, *Journal of Philosophy 71(13)*, 377–391. 9
- Kapteyn, Arie, 1994, The measurement of household cost functions – revealed preference versus subjective measures, *Journal of Population Economics 7*, 333–350. 8
- Kaufman, Dennis A., 1995, Negative externalities and welfare improving preference changes, *Environmental and Resource Economics 6(1)*, July, 53–71. 12
- Leibenstein, Harvey, 1950, Bandwagon, snob, and Veblen effects in the theory of consumers' demand, *Quarterly Journal of Economics 44*, May, 183–207. 10
- Levhari, David and Peles, Yoram, 1973, Market structure, quality and durability, *Bell Journal of Economics 4(1)*, Spring, 235–248. 9
- Musgrave, Richard A., 1959, *The theory of public finance*, McGraw-Hill, Tokyo. 20
- Pollak, Robert A. and Wales, Terence J., 1979, Welfare comparisons and equivalence scales, *American Economic Review 69(2)*, May, 216–221. 11
- Richter, Marcel K., 1971, Rational choice, in: Chipman, John S., Hurwicz, Leonid, Richter, Marcel K., and Sonnenschein, Hugo F. (eds), *Preferences utility and demand - A Minnesota symposium*, Harcourt Brace Jovanovich, New York, 29–58. 4

- Schmalensee, Richard, 1986, Advertising and market structure, in: Stiglitz, Joseph and Matthewson, F. (eds), *New developments in the analysis of market structure*, MIT Press, Cambridge, 373–396. 9
- Sen, Amartya K., 1976, Rational fools, in: Harris, H. (ed), *Scientific models and man: The Herbert Spencer lecture*, Oxford University Press, Oxford. 4
- Sen, Amartya, 1993, Internal consistency of choice, *Econometrica* 61(3), May, 495–521. 4
- Sheshinski, Eytan, 1976, Price, quality and quantity regulation in monopoly situation, *Economica* 43(170), May, 127–137. 9
- Stigler, George J. and Becker, Gary S., 1977, De gustibus non est disputandum, *American Economic Review* 67(2), March, 76–90. 9
- Veblen, Thorstein B., 1899, *The theory of leisure class*. 10
- von Weizsäcker, Carl Christian, 1971, Notes on exogenous changes of tastes, *Journal of Economic Theory* 3(4), December, 345–372. 1, 9
- Yaari, Menahem E., 1978, Endogenous changes in tastes: A philosophical discussion, in: Gottinger, Hans-Werner and Leinfellner, Werner (eds), *Decision Theory and Social Ethics*, Reidel Publishing Co, Dordrecht, 59–98. 9
- Yaari, Menahem E., n.d., Consistent utilization of an exhaustible resource: or how to eat an appetite arousing cake, mimeo. 9

A Social planner

Symbol	Description
s, c, p, m	subscripts indicating the social planner, the consumer, the price-taking firm or the monopolist, respectively
λ	shadow price of the resource/budget constraint
μ	shadow price of upper limit to z
$p_x = 1$	consumer price of the numéraire x
p	consumer price of y
$p_z = 0$	consumer price of z
τ_y	unit tax rate on y
τ_z	unit tax rate on z

Table 1: Notation

The social optimum can be determined by maximizing the Lagrange function

$$\mathcal{L}_s = u[y, z] + x + \lambda_s(\bar{x} - x - c[y, z]) + \mu_s(\bar{z} - z)$$

subject to the non-negativity constraints for x , y , z , λ_s and μ_s . The corresponding first-order conditions are

$$\begin{array}{lll} 1 - \lambda_s \leq 0 & x \geq 0 & x(1 - \lambda_s) = 0 \\ u_y - \lambda_s c_y \leq 0 & y \geq 0 & y(u_y - \lambda_s c_y) = 0 \\ u_z - \lambda_s c_z - \mu_s \leq 0 & z \geq 0 & z(u_z - \lambda_s c_z - \mu_s) = 0 \\ \bar{x} - x - c[y, z] \geq 0 & \lambda_s \geq 0 & \lambda_s(\bar{x} - x - c[y, z]) = 0 \\ \bar{z} - z \geq 0 & \mu_s \geq 0 & \mu_s(\bar{z} - z) = 0. \end{array}$$

For an interior optimum it is necessary for the matrix

$$H_s = \begin{pmatrix} u_{yy} - c_{yy} & u_{yz} - c_{yz} \\ u_{yz} - c_{yz} & u_{zz} - c_{zz} \end{pmatrix}$$

to be negative semi-definite. If no interior solution to the first-order conditions exists or if the second-order condition is violated, then a boundary solution must be optimal.

B Consumer

The optimality conditions for the consumer can be derived from

$$\mathcal{L}_c = u[y, z] + x + \lambda_c(\bar{x} - x - py),$$

leading to the first-order conditions

$$\begin{array}{lll} 1 - \lambda_c \leq 0 & x \geq 0 & x(1 - \lambda_c) = 0 \\ u_y - \lambda_c p \leq 0 & y \geq 0 & y(u_y - \lambda_c p) = 0. \end{array}$$

The necessary condition for an interior optimum is

$$H_c = u_{yy} \leq 0.$$

C Price-taking firm

$$\mathcal{L}_p = py - c[y, z] - \tau_y y - \tau_z z + \mu_p(\bar{z} - z)$$

leads to the first-order conditions

$$p - c_y - \tau_y \leq 0 \quad y \geq 0 \quad y(p - c_y - \tau_y) = 0 \quad (10)$$

$$-c_z - \tau_z - \mu_p \leq 0 \quad z \geq 0 \quad z(-c_z - \tau_z - \mu_p) = 0 \quad (11)$$

$$\bar{z} - z \geq 0 \quad \mu_p \geq 0 \quad \mu_p(\bar{z} - z) = 0. \quad (12)$$

The second-order condition for an interior optimum requires

$$H_p = \begin{pmatrix} -c_{yy} & -c_{yz} \\ -c_{yz} & -c_{zz} \end{pmatrix}$$

to be negative semi-definite which is equivalent to c being convex.

D Monopoly

$$\mathcal{L}_m = u_y y - c[y, z] - \tau_y y - \tau_z z + \mu_m(\bar{z} - z)$$

leads to the first-order conditions

$$u_{yy}y + u_y - c_y - \tau_y \leq 0 \quad y \geq 0 \quad y(u_{yy}y + u_y - c_y - \tau_y) = 0 \quad (13)$$

$$u_{yz}y - c_z - \tau_z - \mu_m \leq 0 \quad z \geq 0 \quad z(u_{yz}y - c_z - \tau_z - \mu_m) = 0 \quad (14)$$

$$\bar{z} - z \geq 0 \quad \mu_m \geq 0 \quad \mu_m(\bar{z} - z) = 0. \quad (15)$$

The Hessian matrix

$$H_m = \begin{pmatrix} u_{yyy}y - 2u_{yy} - c_{yy} & u_{yyz}y - u_{yz} - c_{yz} \\ u_{yyz}y - u_{yz} - c_{yz} & u_{yzz}y - u_{yz} - c_{zz} \end{pmatrix}$$

that has to be negative semi-definite for an interior optimum, involves third-order derivatives of the utility function that are commonly associated with monopoly problems.

E Equilibria and optimality

The above optimality conditions are very general. However, some boundary solutions are without particular interest in the context of the present paper. We will, therefore, assume that the non-negativity constraints for x and y are strictly satisfied. This is equivalent to assuming $u_y[0, z]$ and \bar{x} to be sufficiently large. Boundary solutions with respect to z , on the other hand, arise naturally in the context of variable tastes and contribute significantly to the understanding of allocative and welfare effects. Table 2 allows the comparison of the optimality conditions under different regimes.

It is apparent that interior solutions to the first-order conditions, if they exist, generally differ between the three regimes. Furthermore, they may violate any of the different second-order conditions.

Using Pigouvian-type taxes the government can implement a first-best solution – if the respective second-order conditions of the firm are satisfied. Using the tax rates given in table 3 the solutions to the first-order conditions in all regimes are identical. If the firm behaves competitively, a

Social optimum	Competition	Monopoly
$0 < z < \bar{z}$		
$u_y[y_s^*, z_s^*] = c_y[y_s^*, z_s^*]$ $u_z[y_s^*, z_s^*] = c_z[y_s^*, z_s^*]$	$u_y[y_p^*, z_p^*] = c_y[y_p^*, z_p^*]$ $0 = c_z[y_p^*, z_p^*]$	$u_{yy}[y_m^*, z_m^*]y_m^* + u_y[y_m^*, z_m^*] = c_y[y_m^*, z_m^*]$ $u_{yz}[y_m^*, z_m^*]y_m^* = c_z[y_m^*, z_m^*]$
$z = 0$		
$u_y[y_s, 0] = c_y[y_s, 0]$ $u_z[y_s, 0] \leq c_z[y_s, 0]$	$u_y[y_c, 0] = c_y[y_c, 0]$ $0 \leq c_z[y_c, 0]$	$u_{yy}[y_m, 0]y_m + u_y[y_m, 0] = c_y[y_m, 0]$ $u_{yz}[y_m, 0]y_m \leq c_z[y_m, 0]$
$z = \bar{z}$		
$u_y[\bar{y}_s, \bar{z}] = c_y[\bar{y}_s, \bar{z}]$ $u_z[\bar{y}_s, \bar{z}] \geq c_z[\bar{y}_s, \bar{z}]$	$u_y[\bar{y}_c, \bar{z}] = c_y[\bar{y}_c, \bar{z}]$ $0 \geq c_z[\bar{y}_c, \bar{z}]$	$u_{yy}[\bar{y}_m, \bar{z}]\bar{y}_m + u_y[\bar{y}_m, \bar{z}] = c_y[\bar{y}_m, \bar{z}]$ $u_{yz}[\bar{y}_m, \bar{z}]\bar{y}_m \geq c_z[\bar{y}_m, \bar{z}]$

Table 2: Optimality of solutions for different values of z

Competition	Monopoly
$\tau_z = u_z[y_s^*, z_s^*]$	$\tau_z = u_{yz}[y_s^*, z_s^*]y_s^* + u_z[y_s^*, z_s^*]$
$\tau_y = 0$	$\tau_y = u_{yy}[y_s^*, z_s^*]y_s^*$

Table 3: First-best taxation

single instrument suffices (τ_z). The additional distortion due to monopolistic behaviour requires a second instrument (τ_y).

If the government can levy only one tax, the second-best optimization problem requires the government to optimize the individual's utility subject to the first-order condition of the competitive or monopolistic firm, respectively. If the only tax available is τ_y then the social optimization problem becomes

$$\max_{\tau_y} u[y[\tau_y], z[\tau_y]] - c[y[\tau_y], z[\tau_y]],$$

where $y[\tau_y]$ and $z[\tau_y]$ are defined, implicitly, by the first-order conditions for an interior optimum of a price-taking (10)-(12) or a monopolistic firm (13)-(15), respectively, and $\tau_z = 0$.

Competition	Monopoly
Taxing y only	
$\tau_y = u_z \frac{c_{yz}}{c_{zz}}$	$\tau_y = u_{yy}y + (u_z - c_z) \frac{u_{yyz}y + u_{yz} - c_{yz}}{u_{yzz}y - c_{zz}}$
Taxing z only	
$\tau_z = -u_z$	$\tau_z = -u_z + u_{yz}y + (u_y - c_y) \frac{u_{yyz}y + u_{yz} - c_{yz}}{u_{yyy}y + 2u_{yy} - c_{yy}}$

Table 4: Second-best taxation

The first-order condition

$$(u_y - c_y) \frac{\partial y}{\partial \tau_y} + (u_z - c_z) \frac{\partial z}{\partial \tau_y} = 0$$

can be rewritten for the monopolistic firm using (13) to substitute for $(u_y - c_y)$

$$\tau_y = u_{yy}y - (u_z - c_z) \frac{dz}{dy}.$$

dz/dy can be derived by totally differentiating the firm's first-order condition for z . The optimal tax then becomes

$$\tau_y = u_{yy}y + (u_z - c_z) \frac{u_{yyz}y + u_{yz} - c_{yz}}{u_{yzz}y - c_{zz}}.$$

The optimal tax rates for z or for the case of a competitive firm can be derived in a similar way. They are presented in table 4. In contrast to the first-best taxes these formulae only define the tax rates implicitly because the right-hand side of the equations contains the firm's supply as a function of the tax rate.