Spatially correlated categorical time series: A case study in forest health

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Survey and Data

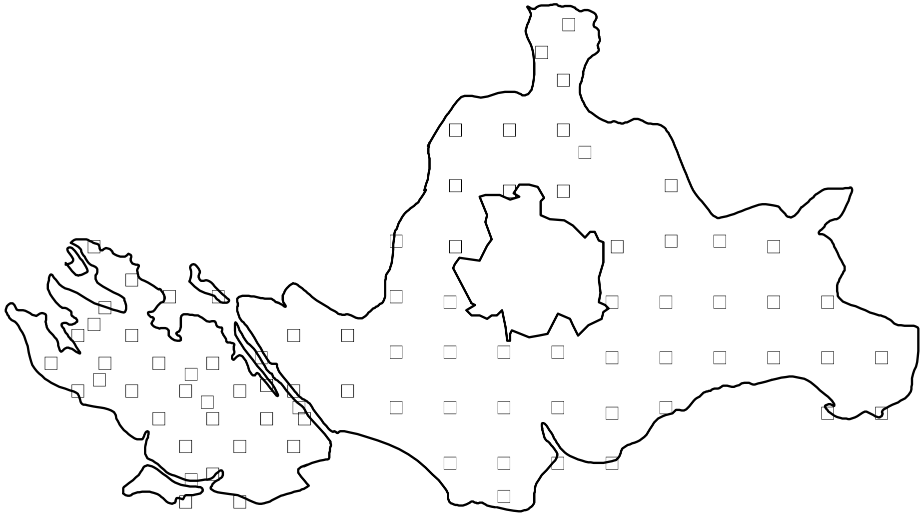
- Aim of the study: Identify factors influencing the health status of trees.

- Database: Yearly visual forest health inventories carried out from 1983 to 2004 in a northern Bavarian forest district.

- 83 observation plots of beeches within a 15 km times 10 km area.

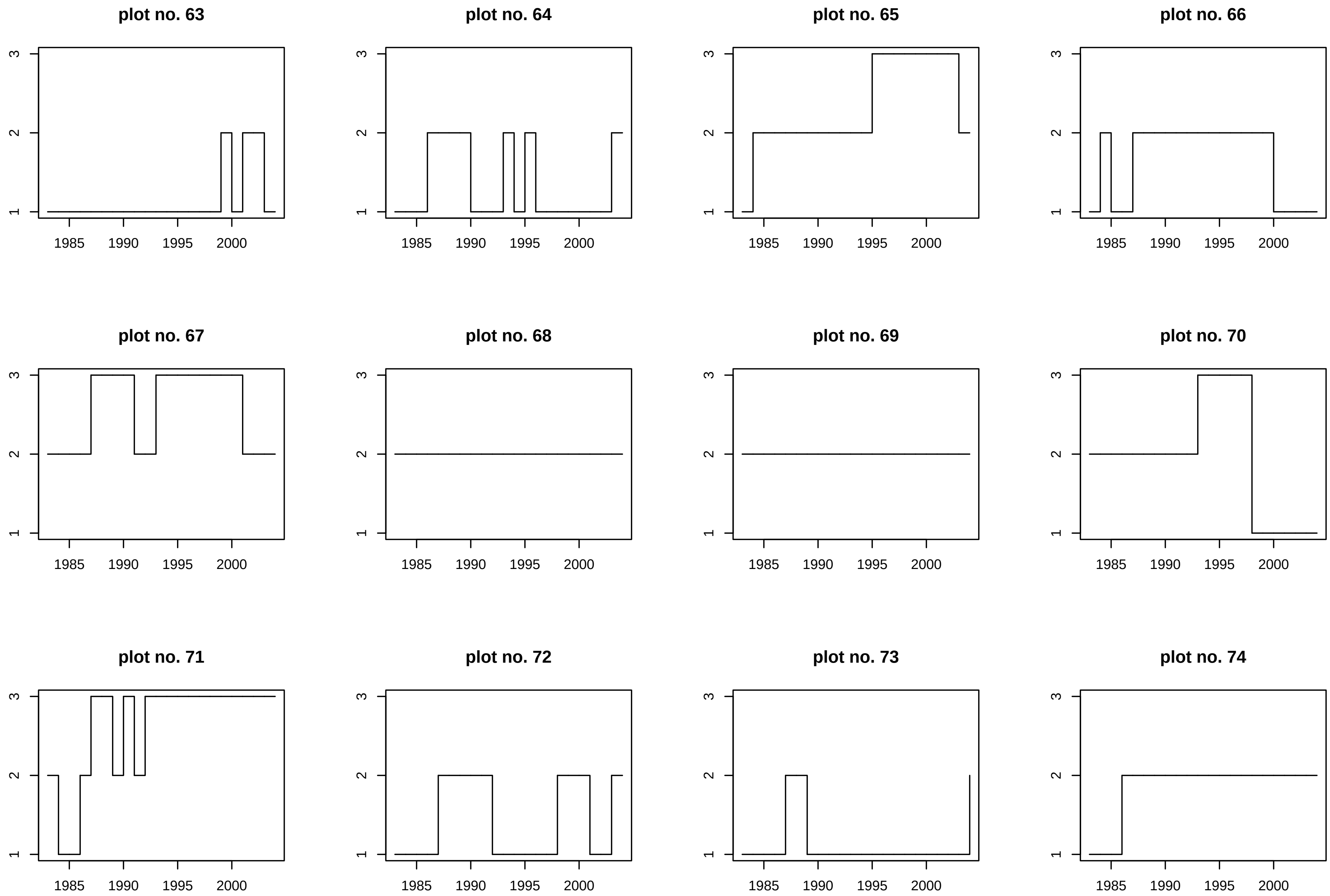
- Response: defoliation degree at plot i in year t, measured in three ordered categories:

$y_{it} = 1$ no defoliation,
$y_{it} = 2$ defoliation 25% or less,
$y_{it} = 3$ defoliation above 25%.



- Covariates:

Continuous:	average age of trees at the observation plot
	elevation above sea level in meters
	inclination of slope in percent
	depth of soil layer in centimeters
	pH-value in 0-2cm depth
	density of forest canopy in percent
Categorical	thickness of humus layer in 5 ordered categories
	level of soil moisture
	base saturation in 4 ordered categories
Binary	type of stand
	application of fertilisation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plot no. 64
plot no. 65
plot no. 66
plot no. 67
plot no. 68
plot no. 69
plot no. 70
plot no. 71
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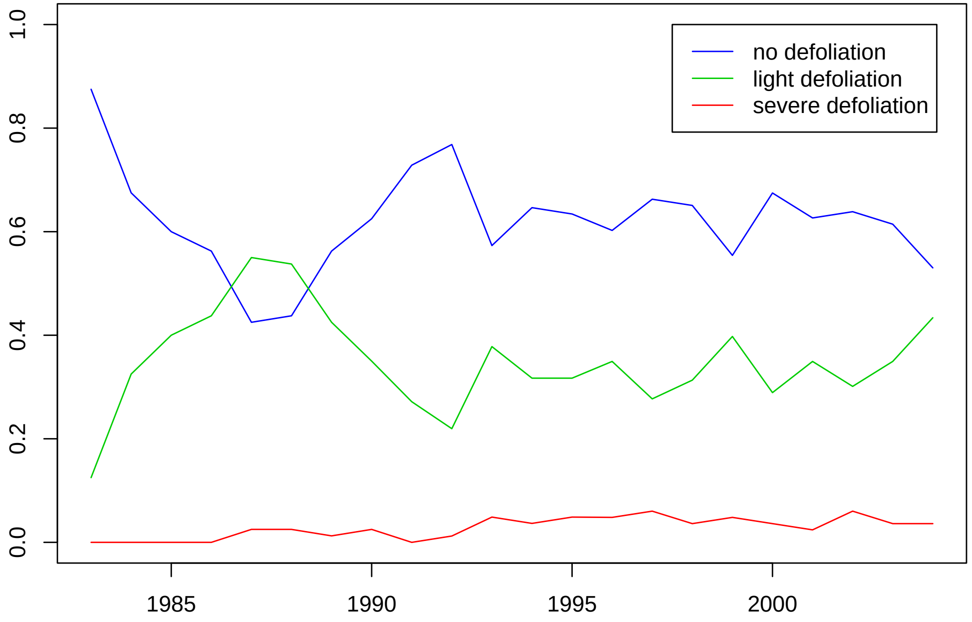
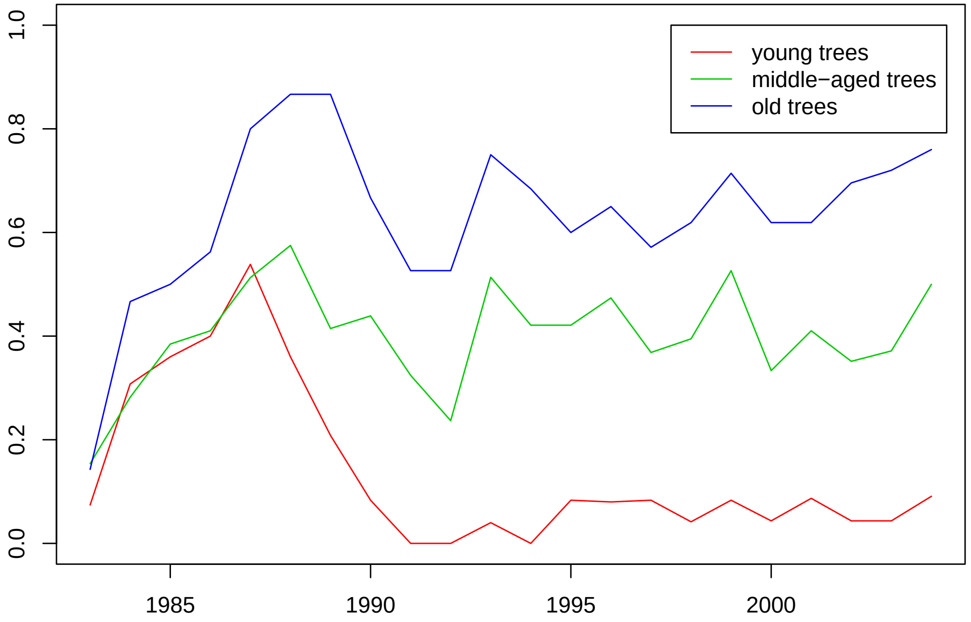
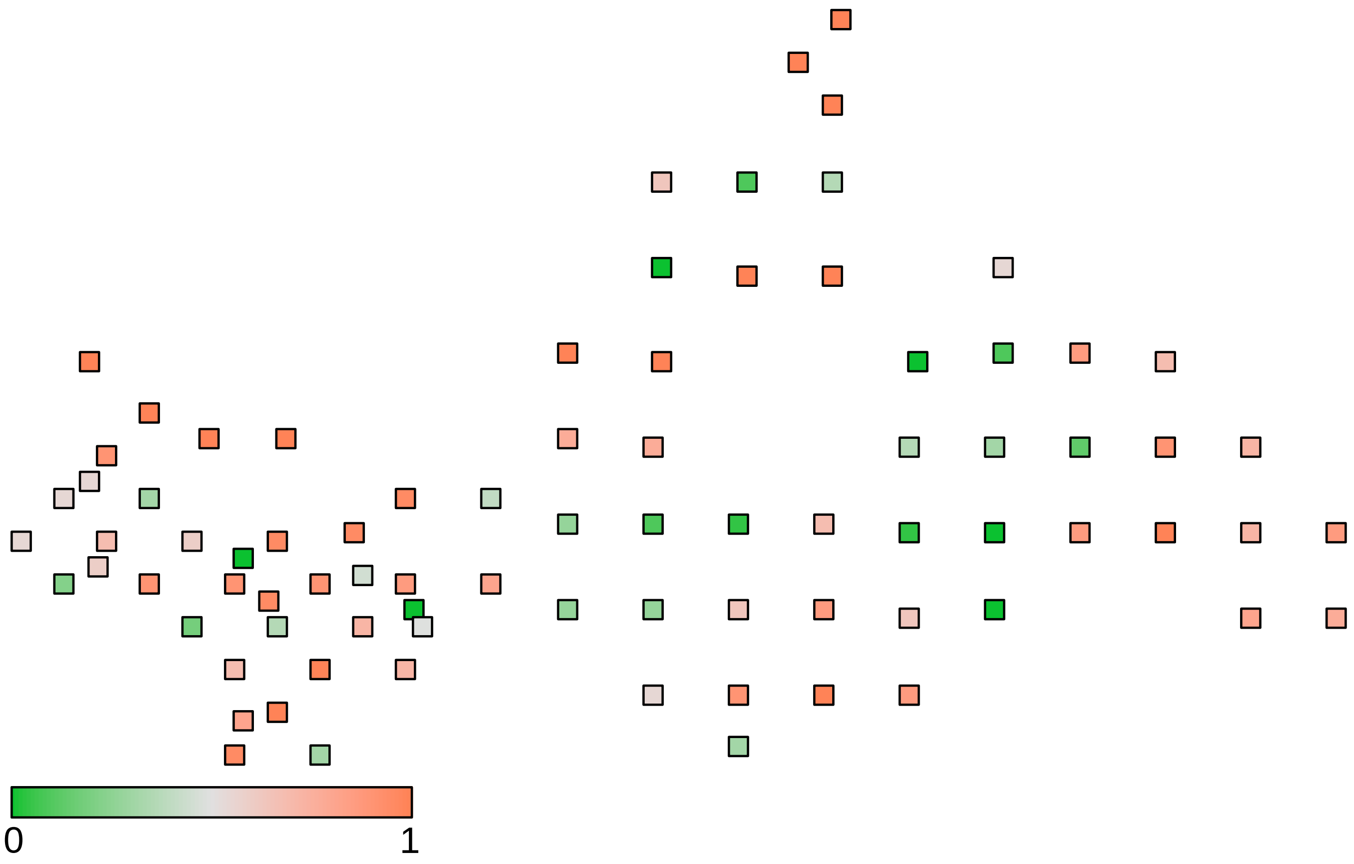
plot no. 73
plot no. 74
1
2
3
1985
1990
1995
2000



Empirical time trends.

Trends for different ages.

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- We need a model that can simultaneously deal with the following issues:
  - A spatially aligned set of time series.

    ⇒ Both spatial and temporal correlations have to be considered.
  - Decide whether unobserved heterogeneity is spatially structured or not.
  - Non-linear effects of continuous covariates (e.g. age).
  - A possibly time-varying effect of age (i.e. an interaction between age and calendar time).
  - A categorical response variable.

Regression models for ordinal responses

- Defoliation degree is measured in three ordered categories.
- Derive regression models for ordinal responses based on latent variables:

$$D = x'\beta + \varepsilon.$$

- D can be considered an unobserved, continuous measure of defoliation.
- Link D to the categorical response Y based on ordered thresholds

$$-\infty = \theta^{(0)} < \theta^{(1)} < \theta^{(2)} < \theta^{(3)} = \infty$$

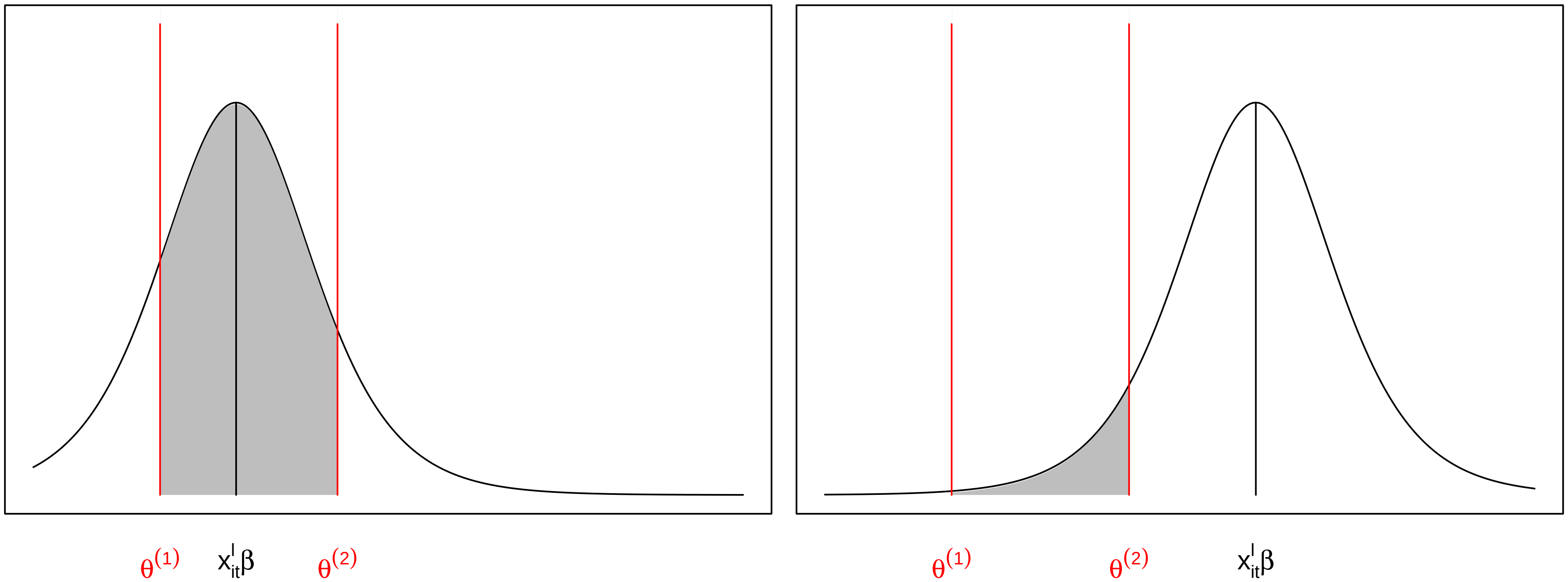
via

$$Y = r \quad \Leftrightarrow \quad \theta^{(r-1)} < D \leq \theta^{(r)}.$$

- Defines cumulative probabilities in terms of the cdf F of the latent error term ε:

$$P(Y \leq r) = P(D \leq \theta^{(r)}) = P(x'\beta + \varepsilon \leq \theta^{(r)}) = F(\theta^{(r)} - x'\beta).$$

- Intuitive interpretation:



- The thresholds slice the density $f = F'$.

- Three main concepts to account for the longitudinal structure:
  - Marginal models (define working correlations, short time series),
  - Autoregressive models (include lagged response variables as predictors, prediction),
  - Models with random effects

$$D_{it} = x_{it}'\beta + z_{it}'b_i + \varepsilon_{it}.$$

- In the forest health example:
  - Relatively long series.
  - Interest is on modelling the marginal expectation, not the conditional expectation.
  - In addition: spatial correlations, non-linear trends, further non-linear effects.

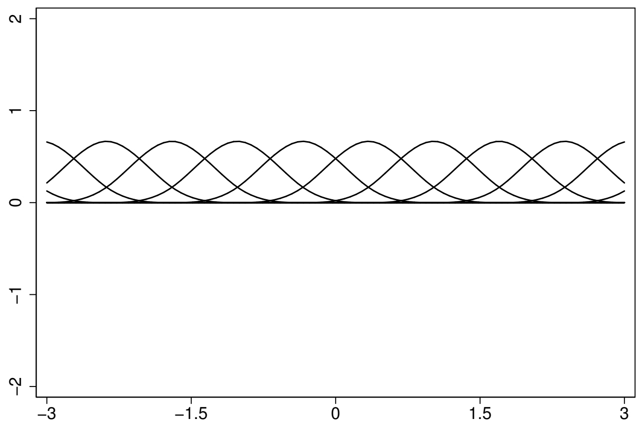
$\Rightarrow$ Extend mixed models to geoadditive mixed models.

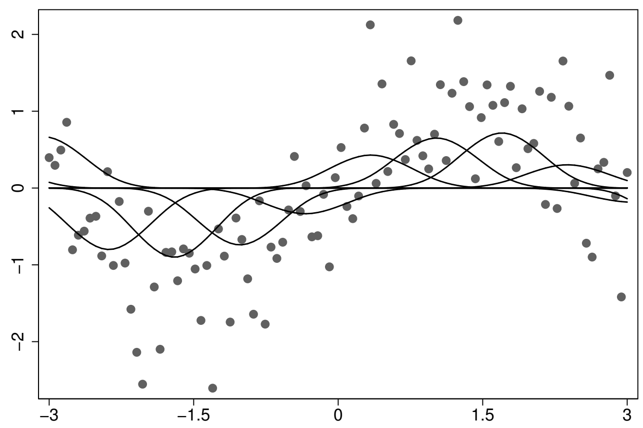
Geoadditive mixed models

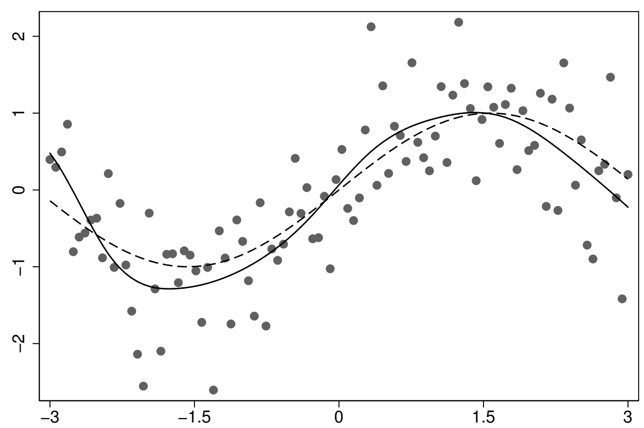
- Suitable model in our application:

$$
\begin{aligned}
D_{it} \quad = \quad & f_1(age_{it}) && \text{nonlinear effects of age,} \\
& +f_2(inc_i) && \text{inclination of slope, and} \\
& +f_3(can_{it}) && \text{canopy density.} \\
& +f_{time}(t) && \text{nonlinear time trend.} \\
& +f_4(t, age_{it}) && \text{interaction between age and calendar time.} \\
& +f_{spat}(s_i) && \text{structured and} \\
& +b_i && \text{unstructured spatial random effects.} \\
& +x'_{it}\gamma && \text{usual parametric effects.} \\
& +\varepsilon_{it} && \text{error term.}
\end{aligned}
$$

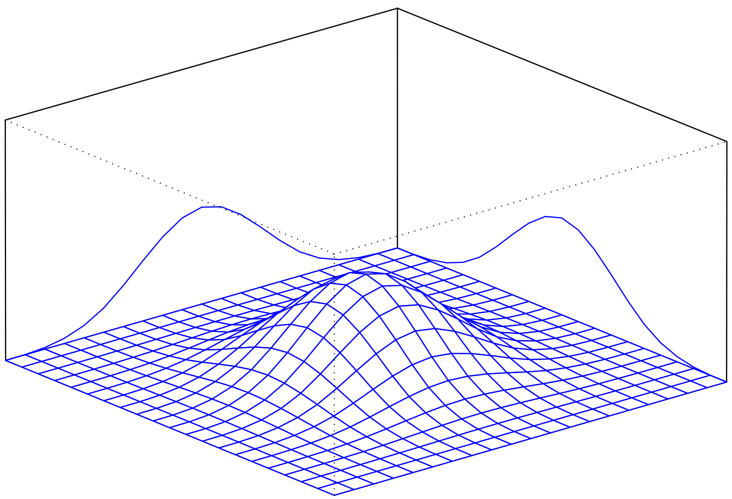
- Penalised splines: Nonlinear covariate effects, nonlinear time trends.
  - Approximate $f(x)$ by a weighted sum of B-spline basis functions.
  - Employ a large number of basis functions to enable flexibility.
  - Penalise differences between parameters of adjacent basis functions to ensure smoothness.

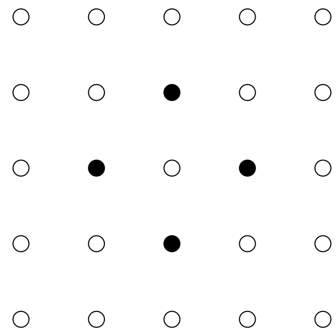




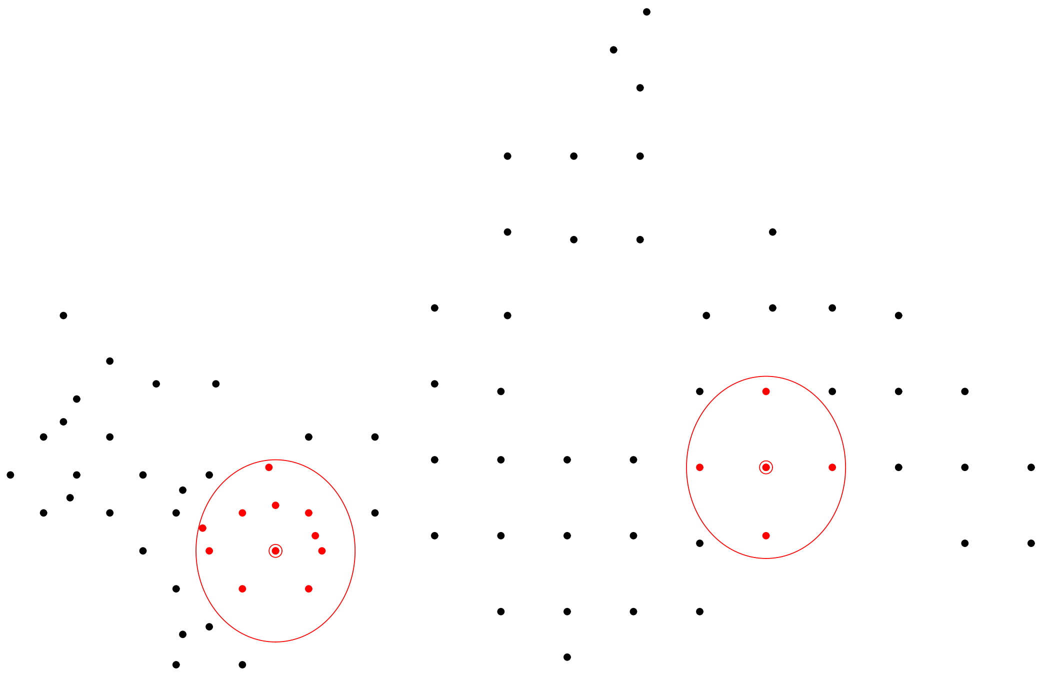


- Bivariate penalised splines: Interaction surfaces, structured spatial effect.
  - Bivariate basis functions based on tensor product B-splines.
  - Extend penalisation to neighbours on a grid.

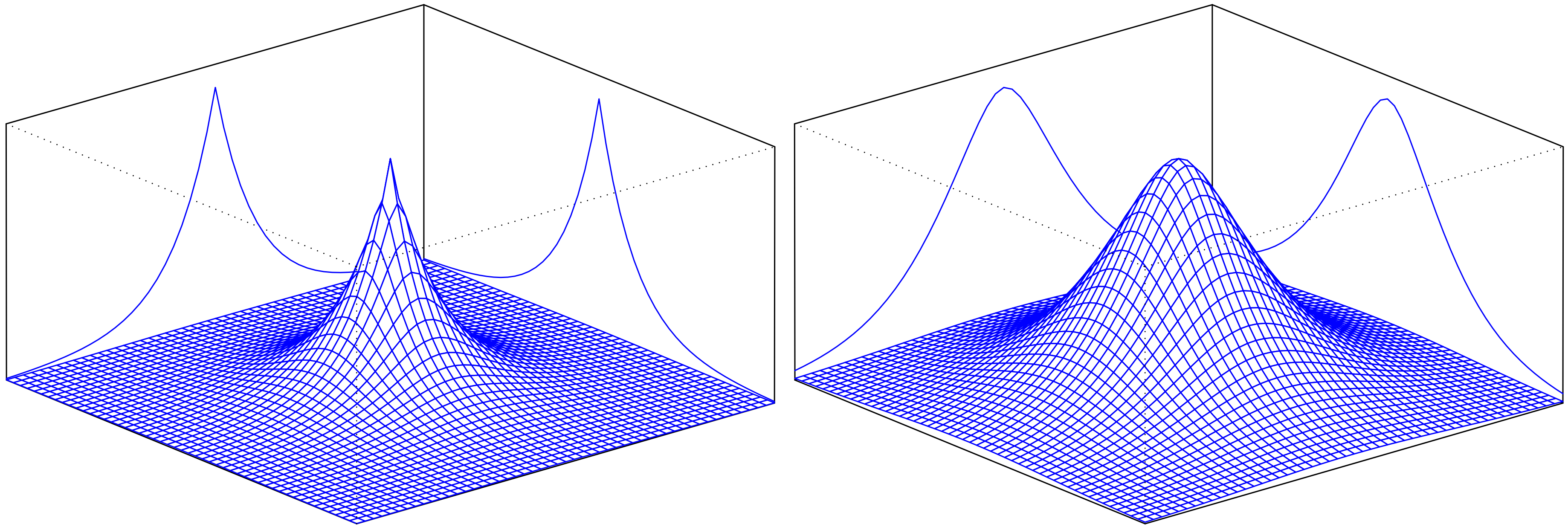




- Markov random fields: Structured spatial effect.
  - Bivariate extension of a first order random walk on the real line.
  - Define two observation plots as neighbours if their distance is less than 1.2km.
  - Assume that the expected value of $f_{spat}(s)$ is the average of the function evaluations of adjacent sites.



- Stationary Gaussian random fields: Structured spatial effect.
  - Well-known as Kriging in the geostatistics literature.
  - Spatial effect follows a zero mean stationary Gaussian stochastic process.
  - Correlation of two arbitrary sites is defined by an intrinsic correlation function.



Mixed model based inference

- Each term in the predictor is associated with a vector of regression coefficients with improper multivariate Gaussian prior:

$$p(\beta_j|\tau_j^2) \propto \exp\left(-\frac{1}{2\tau_j^2}\beta_j' K_j \beta_j\right)$$

⇒ Reparametrize the model to a proper mixed model.

- Obtain empirical Bayes estimates via iterating
  - Penalized maximum likelihood for regression coefficients.
  - Restricted Maximum / Marginal likelihood for variance parameters.

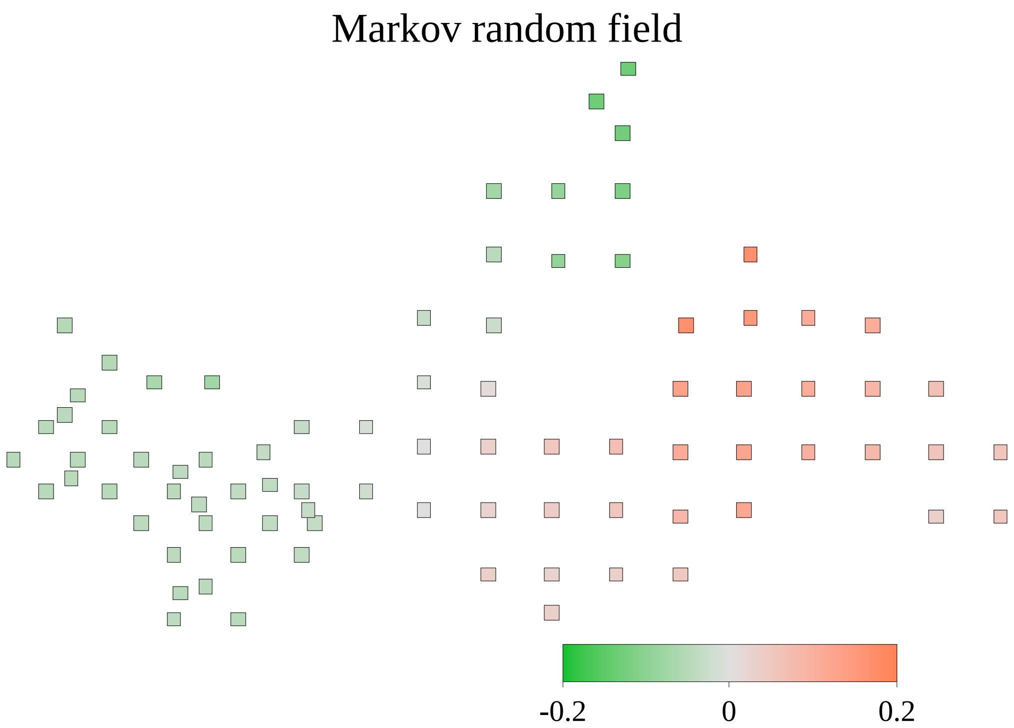
Software

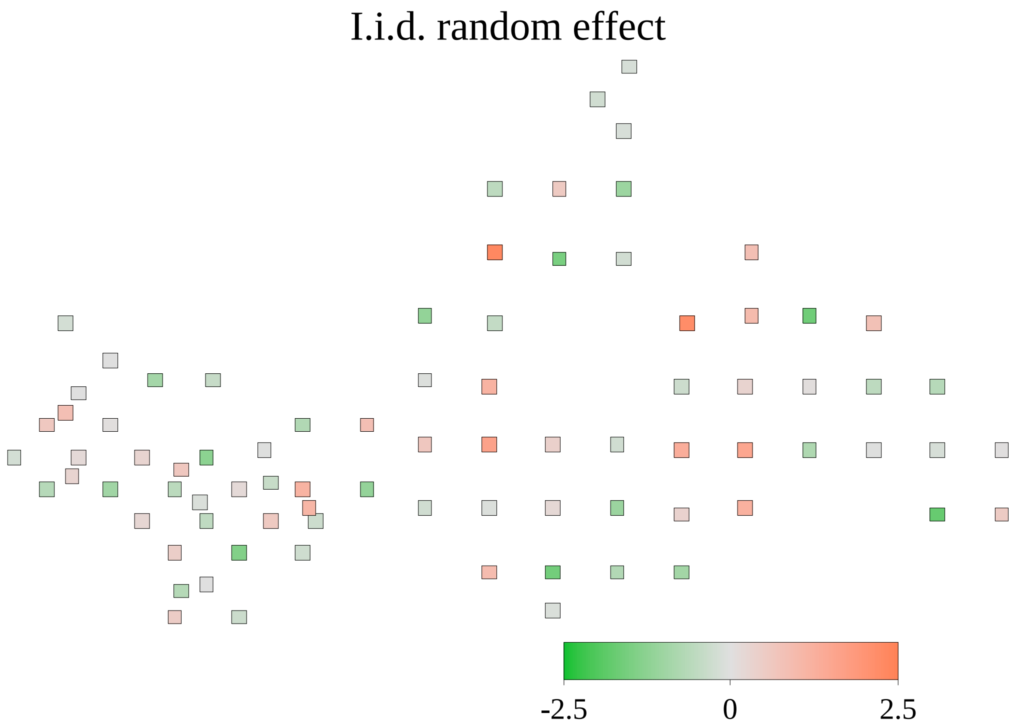
- Implemented in the software package BayesX.

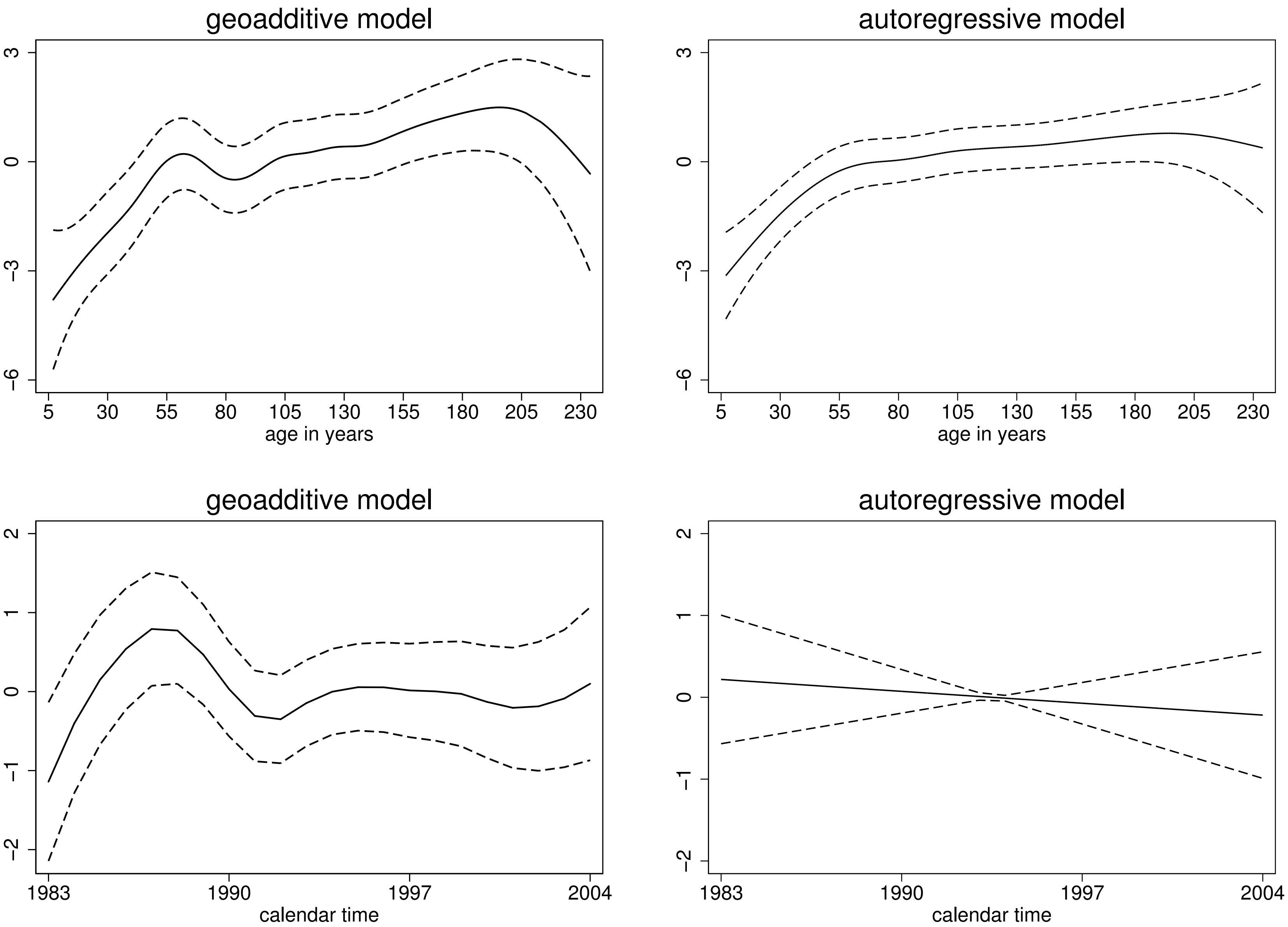
- Available from

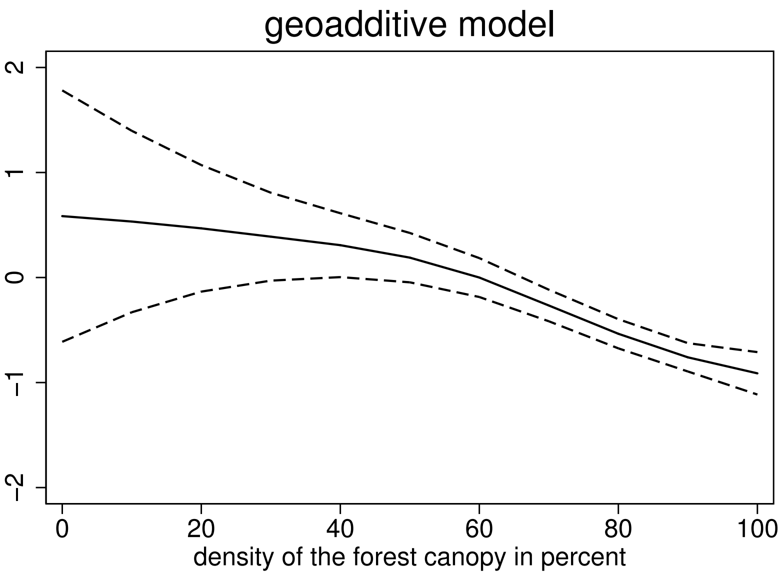
`http://www.stat.uni-muenchen.de/~bayesx`

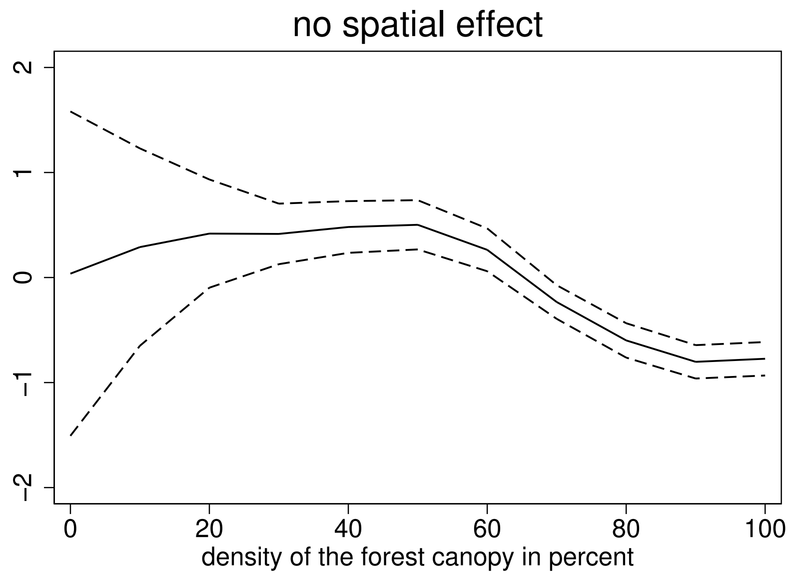
Results

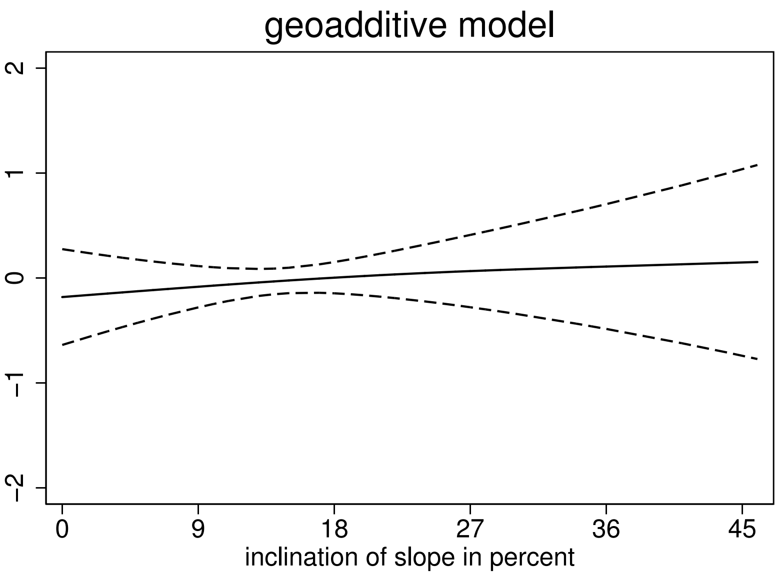


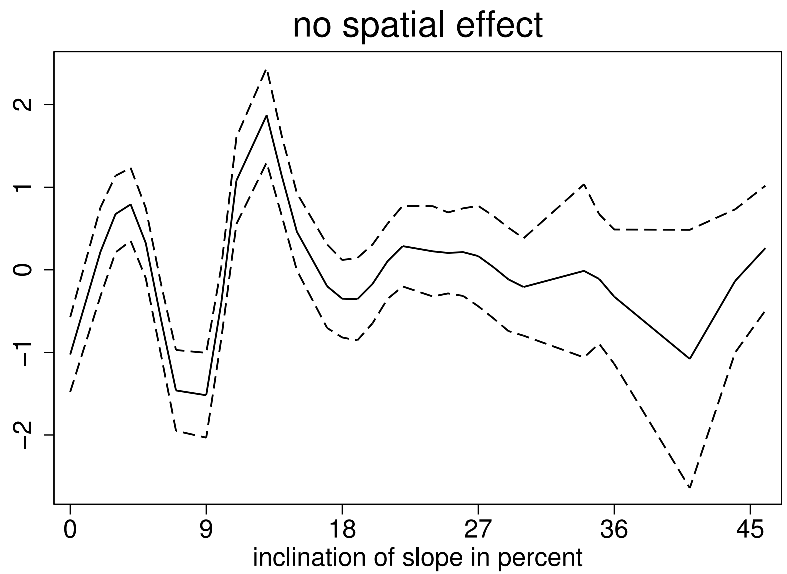



geoadditive model
autoregressive model
3
0
−3
−6
5 30 55 80 105 130 155 180 205 230
age in years
geoadditive model
autoregressive model
2
1
0
−1
−2
1983 1990 1997 2004
calendar time


geoadditive model
−2
−1
0
1
2
0
20
40
60
80
100
density of the forest canopy in percent


no spatial effect
−2
−1
0
1
2
0
20
40
60
80
100
density of the forest canopy in percent


geoadditive model
−2
−1
0
1
2
0
9
18
27
36
45
inclination of slope in percent


no spatial effect
−2
−1
0
1
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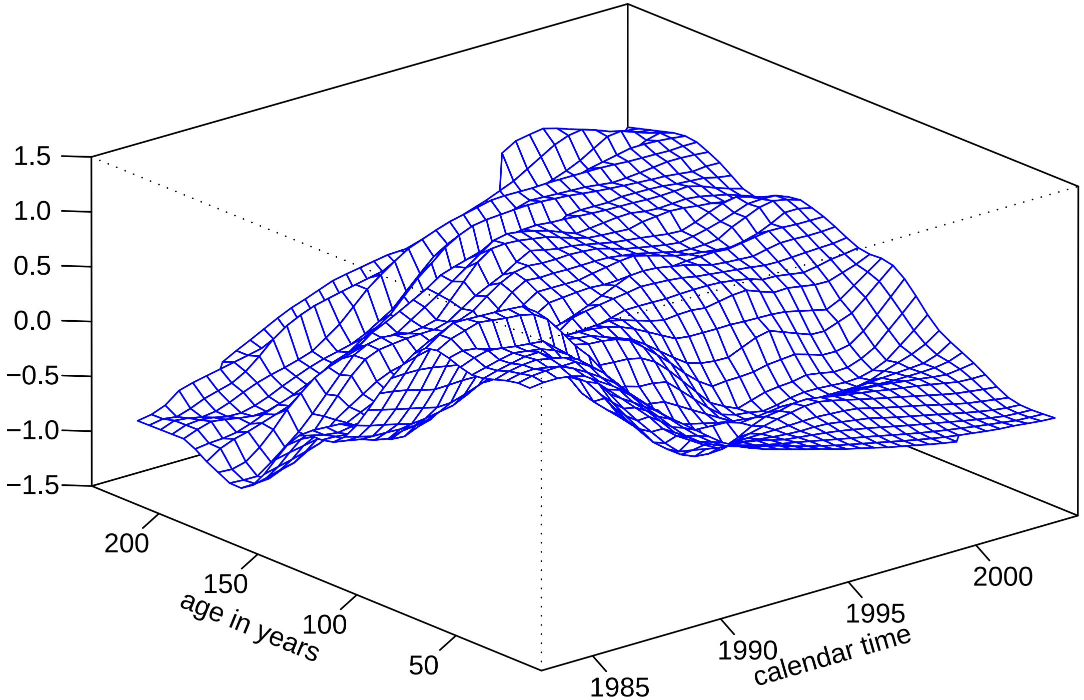
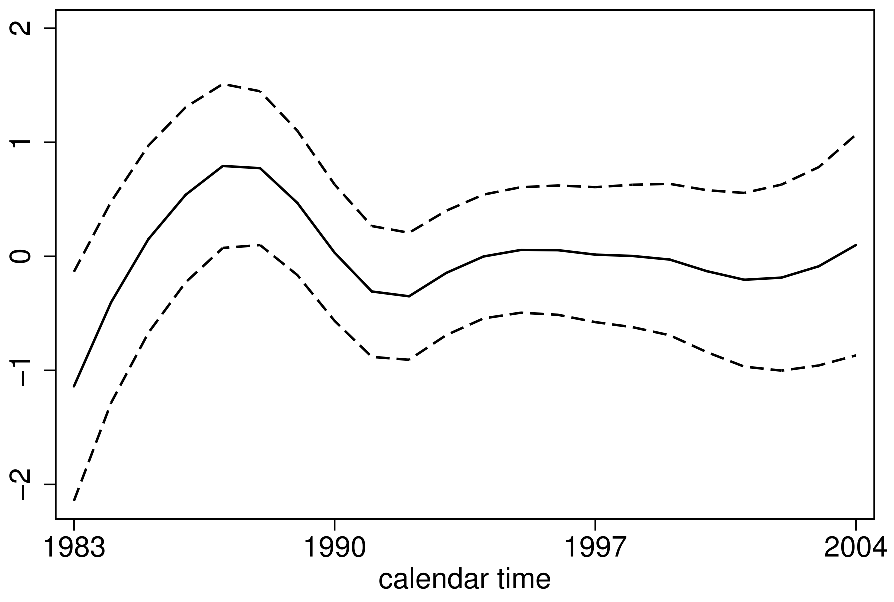
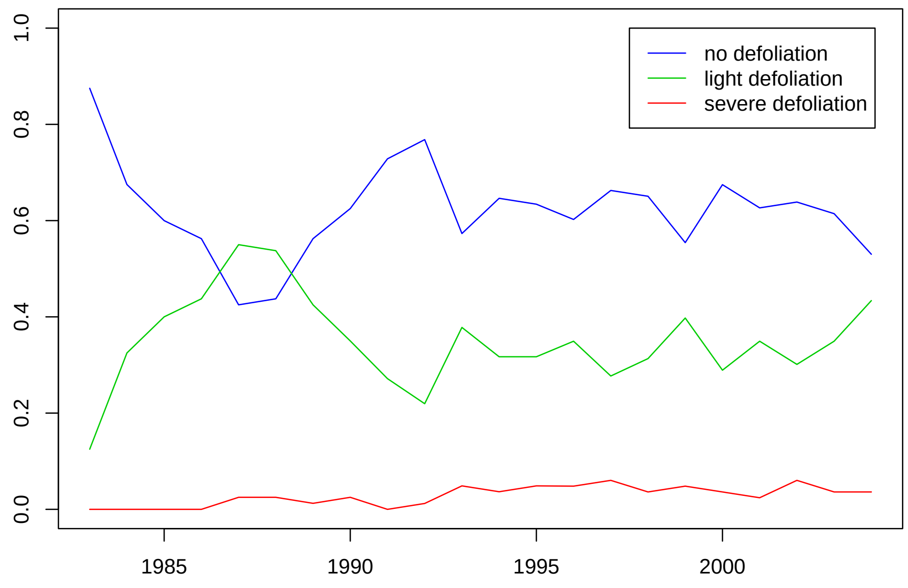
9
18
27
36
45
inclination of slope in percent



variable	$\hat{\beta}_j$	std. dev.	p-value
ph	-0.037	0.212	0.860
humus 0-1cm	-0.261	0.108	0.015
humus 1-2cm	-0.135		
humus 2-3cm	0.139	0.086	0.105
humus 3-4cm	0.135	0.102	0.185
humus >4cm	0.122	0.142	0.391
moderately dry	-0.597	0.320	0.061
moderately moist	0.185		
moist or temporary wet	0.412	0.229	0.071

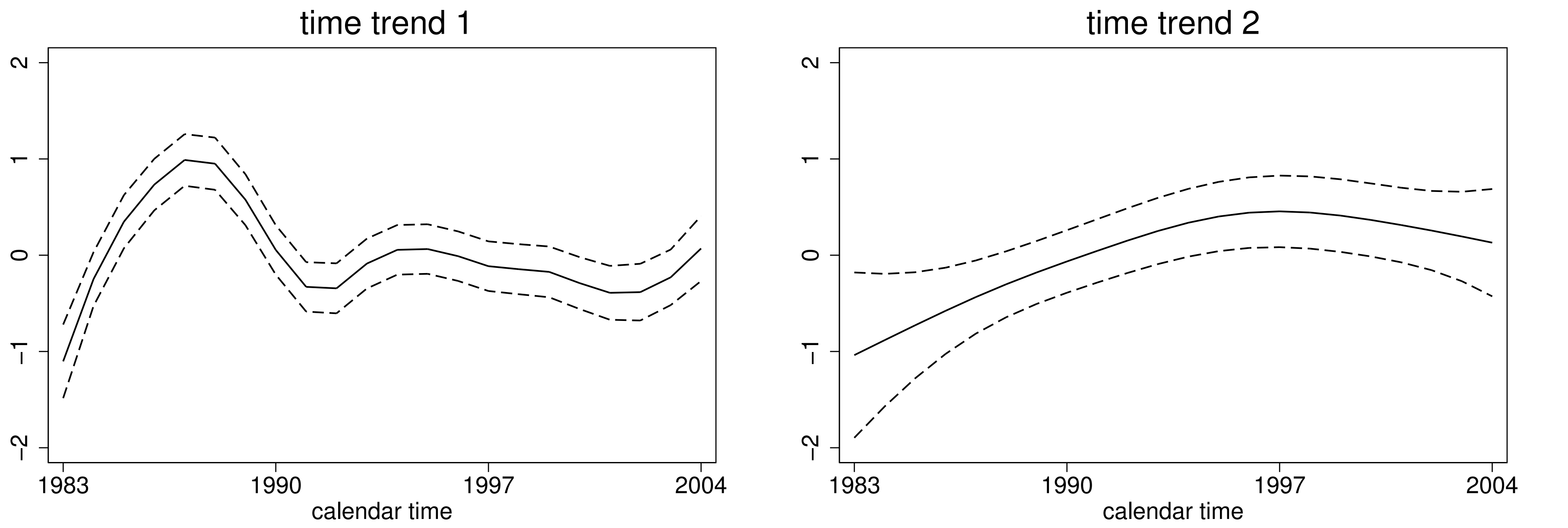
- Limitation of the model: All effects are globally defined





- Possible refinement: Category-specific trends

$$P(Y_{it} \leq r) = \Phi\left[\theta^{(r)} - \ldots - f_{time}^{(r)}(t) - \ldots\right]$$



- More complicated constraints:

$$\theta^{(1)} - f^{(1)}_{time}(t) < \theta^{(2)} - f^{(2)}_{time}(t) \qquad \text{for all } t.$$

Conclusions

- Inclusion of any kind of spatial effect leads to a dramatically improved model fit.
- The unstructured part dominates the structured spatial effect.
- Nonparametric effects allow for more realistic models.
- Category-specific effects give additional insight but may require a larger database.

References

- Kneib, T. & Fahrmeir, L. (2006): Structured additive regression for categorical space-time data: A mixed model approach. *Biometrics*, to appear.

- Kneib, T. & Fahrmeir, L. (2007): A Space-Time Study on Forest Health. In: Chandler, R. E. & Scott, M. (eds.): Statistical Methods for Trend Detection and Analysis in the Environmental Sciences, Wiley.

- A place called home:

`http://www.stat.uni-muenchen.de/~kneib`