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Letter to the editor

The editors received the following note pointing out a gap in the proof of the paper:
"On the Toeplitz pencil conjecture" by M.C. Gouveia, LAMA Vol. 61, Issue 6, 2013.
Stephen Kirkland and Chi-Kwong Li

Comments on some arguments in the article:
"On the Toeplitz pencil conjecture" by M.C. Gouveia, LAMA Vol. 61, Issue 6, 2013.
Wiland Schmale, Oldenburg

I will comment on the arguments which are used to show the vanishing of the limit in formulae (13) under several specific assumptions. These arguments are decisive for the approach chosen to prove the conjecture.

y is taken from the set

$$\mathcal{R} \cap \{z \in \mathbb{C} : \mathsf{rank}(T(z)) = n-3\} \cap \{z \in \mathbb{C} : v_1(z) = 0\}.$$

$\mathcal{R}$ is constructed shortly after formula (8).

In addition there are the following overall assumptions:

$$c_1 \neq 0, \ldots, c_{n-2} \neq 0 \quad \text{and:} \quad (v_1, \ldots, v_{n-2}) \text{ is a unimodular vector from } \mathbb{C}[x]^{n-2}.$$

So, automatically: $v_2(y) \neq 0$ and $v_3(y) \neq 0$ and by construction of $\mathcal{R}$: $y \neq 0$.

In this scenario Gouveia wants to show that the limit in (13) is zero.

But, the denominator $\frac{1}{c_2}\left(c_1 + x\frac{v_3}{v_2}\right)$ can become zero at y depending on the parameters c_i, since $y\frac{v_3(y)}{v_2(y)} \neq 0$ and at the same time also $\frac{1}{y}\, c_1 \neq 0$.

A similar effect will be present in the parallel case still to be considered for a zero of v_2, which is not treated.

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