Beispiel 17.8 (b)

```
> restart:
```

Ein Polynom, das durch iteriertes Wurzelziehen auflösbar ist:

```
f:=x^12-10*x^6+1;
```

$$f := x^{12} - 10\,x^6 + 1$$

Ist f unzerlegbar ?

```
> factor(f);
```

$$x^{12} - 10\,x^6 + 1$$

Maple sagt demnach "ja".
Dass Maple dies so schnell entscheiden kann beruht auf Theorie und Verfahren des entsprechenden Teilgebietes der sogenannten *Computeralgebra*.Ein Standardwerk dazu ist das Buch *"Modern Computer Algebra" von Joachim von zur Gathen und Jürgen Gerhard*.Die Internetseite zum Buch ist: Modern Computer Algebra .

```
> factor(f,sqrt(2));
```

$$(x^6 - 2\,x^3\sqrt{2} - 1)\,(x^6 + 2\,x^3\sqrt{2} - 1)$$

```
> factor(f,sqrt(3));
```

$$(x^6 + 2\,x^3\sqrt{3} + 1)\,(x^6 - 2\,x^3\sqrt{3} + 1)$$

```
> factor(f,{sqrt(2),sqrt(3)});
```

$$(x^3 - \sqrt{3} - \sqrt{2})\,(x^3 + \sqrt{3} - \sqrt{2})\,(x^3 - \sqrt{3} + \sqrt{2})\,(x^3 + \sqrt{3} + \sqrt{2})$$

```
> alias(alpha=(sqrt(2)+sqrt(3))^(1/3));
```

$$\alpha$$

```
> ff:=simplify(factor(f,{sqrt(2),sqrt(3),alpha}));
```

$$ff := (x^2 + x\,\alpha + (\sqrt{3} + \sqrt{2})^{(2/3)})\,(x^2 - (\sqrt{3} + \sqrt{2})^{(2/3)}\,x\sqrt{3} + (\sqrt{3} + \sqrt{2})^{(2/3)}\,x\sqrt{2} + \alpha\sqrt{3} - \alpha\sqrt{2})$$
$$(x^2 + (\sqrt{3} + \sqrt{2})^{(2/3)}\,x\sqrt{3} - (\sqrt{3} + \sqrt{2})^{(2/3)}\,x\sqrt{2} + \alpha\sqrt{3} - \alpha\sqrt{2})\,(x^2 - x\,\alpha + (\sqrt{3} + \sqrt{2})^{(2/3)})$$
$$(x + (\sqrt{3} + \sqrt{2})^{(2/3)}\sqrt{3} - (\sqrt{3} + \sqrt{2})^{(2/3)}\sqrt{2})\,(x - (\sqrt{3} + \sqrt{2})^{(2/3)}\sqrt{3} + (\sqrt{3} + \sqrt{2})^{(2/3)}\sqrt{2})$$
$$(x^2 - (\sqrt{3} + \sqrt{2})^{(2/3)})$$

```
> g:=op(1,ff);h:=op(2,ff);k:=op(3,ff); l:=op(4,ff);
```

$$g := x^2 + x\,\alpha + (\sqrt{3} + \sqrt{2})^{(2/3)}$$

$$h := x^2 - (\sqrt{3} + \sqrt{2})^{(2/3)}\,x\sqrt{3} + (\sqrt{3} + \sqrt{2})^{(2/3)}\,x\sqrt{2} + \alpha\sqrt{3} - \alpha\sqrt{2}$$

$$k := x^2 + (\sqrt{3} + \sqrt{2})^{(2/3)}\,x\sqrt{3} - (\sqrt{3} + \sqrt{2})^{(2/3)}\,x\sqrt{2} + \alpha\sqrt{3} - \alpha\sqrt{2}$$

$$l := x^2 - x\,\alpha + (\sqrt{3} + \sqrt{2})^{(2/3)}$$

```
> collect(h,x);
```

$$x^2 + (-(\sqrt{3} + \sqrt{2})^{(2/3)}\sqrt{3} + (\sqrt{3} + \sqrt{2})^{(2/3)}\sqrt{2})\,x - \alpha\sqrt{2} + \alpha\sqrt{3}$$

Maple erkennt nicht α^2 in diesen Ausdrücken ! Die Nullstellen des letzten quadratischen Polynoms sind laut Maple:

```
> so:=[solve(%)]:so[1];so[2];
```

$$\frac{(\sqrt{3} + \sqrt{2})^{(2/3)}\sqrt{3}}{2} - \frac{(\sqrt{3} + \sqrt{2})^{(2/3)}\sqrt{2}}{2}$$
$$+ \frac{1}{2}\,I\,\sqrt{-5\,(\sqrt{3} + \sqrt{2})^{(4/3)} + 2\,(\sqrt{3} + \sqrt{2})^{(4/3)}\sqrt{3}\sqrt{2} - 4\,\alpha\sqrt{2} + 4\,\alpha\sqrt{3}}$$

$$\frac{(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{3}}{2}-\frac{(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{2}}{2}$$
$$-\frac{1}{2}I\sqrt{-5(\sqrt{3}+\sqrt{2})^{(4/3)}+2(\sqrt{3}+\sqrt{2})^{(4/3)}\sqrt{3}\sqrt{2}-4\alpha\sqrt{2}+4\alpha\sqrt{3}}$$

In diesen Ausdrücken taucht die komplexe Zahl i (bei Maple I) auf. Dass komplexe Lösungen vorliegen hätte man besser vorher erkundet. Mit dieser Information ergibt sich nämlich wie folgt eine vollständige Zerlegung in Linearfaktoren:

```
> ff:=simplify(factor(f,{sqrt(2),sqrt(3),alpha,I}));
```

$$ff := (2x+\alpha-\alpha\sqrt{3}I)(2x+\alpha+\alpha\sqrt{3}I)$$
$$(2x-(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{3}+(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{2}+\sqrt{3}\sqrt{2}(\sqrt{3}+\sqrt{2})^{(2/3)}I-3I(\sqrt{3}+\sqrt{2})^{(2/3)})$$
$$(2x-(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{3}+(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{2}+3I(\sqrt{3}+\sqrt{2})^{(2/3)}-\sqrt{3}\sqrt{2}(\sqrt{3}+\sqrt{2})^{(2/3)}I)$$
$$(2x+(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{3}-(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{2}+\sqrt{3}\sqrt{2}(\sqrt{3}+\sqrt{2})^{(2/3)}I-3I(\sqrt{3}+\sqrt{2})^{(2/3)})$$
$$(2x+(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{3}-(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{2}+3I(\sqrt{3}+\sqrt{2})^{(2/3)}-\sqrt{3}\sqrt{2}(\sqrt{3}+\sqrt{2})^{(2/3)}I)$$
$$(2x-\alpha+\alpha\sqrt{3}I)(2x-\alpha-\alpha\sqrt{3}I)(x+(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{3}-(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{2})$$
$$(x-(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{3}+(\sqrt{3}+\sqrt{2})^{(2/3)}\sqrt{2})(x^2-(\sqrt{3}+\sqrt{2})^{(2/3)})/256$$

Teilprobe durch Einsetzen der zuerst aufgeführten Nullstelle:

```
> simplify(subs(x=-(alpha+I*alpha*sqrt(3))/2,f));
```

$$0$$

An den vorangegangen Rechnung erkennt man dass ein Zerfällungskörper von f über Q den Grad 24 hat. Dies kann man auch wie folgt bestätigen:

```
> alias(beta=RootOf(f));
```

$$\alpha, \beta$$

```
> factor(f,beta);
```

$$(x^2-10x\beta^5+x\beta^{11}+10\beta^4-\beta^{10})(x^2+10x\beta^5-x\beta^{11}+10\beta^4-\beta^{10})(x^2-x\beta+\beta^2)(x^2+x\beta+\beta^2)$$
$$(x-10\beta^5+\beta^{11})(x+10\beta^5-\beta^{11})(x+\beta)(x-\beta)$$

```
> factor(f,{beta,RootOf(x^2+10*x*beta^5-x*beta^11+10*beta^4-beta^10)});
```

$$(x+\mathrm{RootOf}(\_Z^2+(10\beta^5-\beta^{11})\_Z-\beta^{10}+10\beta^4))$$
$$(-10\beta^5+\beta^{11}-\mathrm{RootOf}(\_Z^2+(10\beta^5-\beta^{11})\_Z-\beta^{10}+10\beta^4)+x)$$
$$(10\beta^5-\beta^{11}+\mathrm{RootOf}(\_Z^2+(10\beta^5-\beta^{11})\_Z-\beta^{10}+10\beta^4)+x)$$
$$(x-\mathrm{RootOf}(\_Z^2+(10\beta^5-\beta^{11})\_Z-\beta^{10}+10\beta^4))(x-10\beta^5+\beta^{11})(x+10\beta^5-\beta^{11})$$
$$(-\beta-\mathrm{RootOf}(\_Z^2+(10\beta^5-\beta^{11})\_Z-\beta^{10}+10\beta^4)\beta^2+x)$$
$$(\mathrm{RootOf}(\_Z^2+(10\beta^5-\beta^{11})\_Z-\beta^{10}+10\beta^4)\beta^2+x)$$
$$(-\mathrm{RootOf}(\_Z^2+(10\beta^5-\beta^{11})\_Z-\beta^{10}+10\beta^4)\beta^2+x)$$
$$(\beta+\mathrm{RootOf}(\_Z^2+(10\beta^5-\beta^{11})\_Z-\beta^{10}+10\beta^4)\beta^2+x)(x+\beta)(x-\beta)$$

```
>
>
>
```